Operations & Supply Chain University

Chapter 12: INVENTORY MANAGEMENT

Chapter 12 Learning Outcomes:

- Conduct an ABC analysis
- Explain and use cycle counting
- Explain and use the EOQ and EPQ models for independent inventory demand
- Compute a reorder point and safety stock
- Explain and use the quantity discount model

Managing Inventories

Operations & Supply Chain University

- Inventory: any asset held for future use or sale
  - Expenses of financing and maintaining inventories are a substantial part of the cost of doing business

- Inventory Management: involves planning, coordinating, and controlling the acquisition, storage, handling, movement, distribution, and possible sale of raw materials, component parts and subassemblies, supplies and tools, replacement parts, and other assets that are needed to meet customer wants and needs
  - The objective is to strike a balance between inventory investment and customer service

Functions of Inventory

- Inventory can serve several functions that add flexibility to a firm's operations
  1. To provide a selection of goods for anticipated customer demand and to separate the firm from fluctuations in that demand
  2. To decouple various parts of the production process
  3. To take advantage of quantity discounts
  4. To hedge against inflation

Types of Inventory

RM	WIP	FG
Raw Material	Work In Process	Finished Goods
(Input)	(Transformation/Value Add)	(Output)

- Raw materials (RM): component parts, subassemblies, and supplies are inputs to manufacturing and service-delivery processes
- Work-in-process (WIP): partially finished products in various stages of completion that are awaiting further processing
  - Cycle time: the time it takes for a product to be made
- Finished goods (FG): completed products ready for distribution or sale to customers

Types of Inventory

Raw Material
(Input)

Work In Process
(Transformation/Value Add)

Finished Goods
(Output)

Types of Inventory

Operations & Supply Chain University

Raw Material
(Input)

Work In Process
(Transformation/Value Add)

Finished Goods
(Output)

Types of Inventory

Operations & Supply Chain University

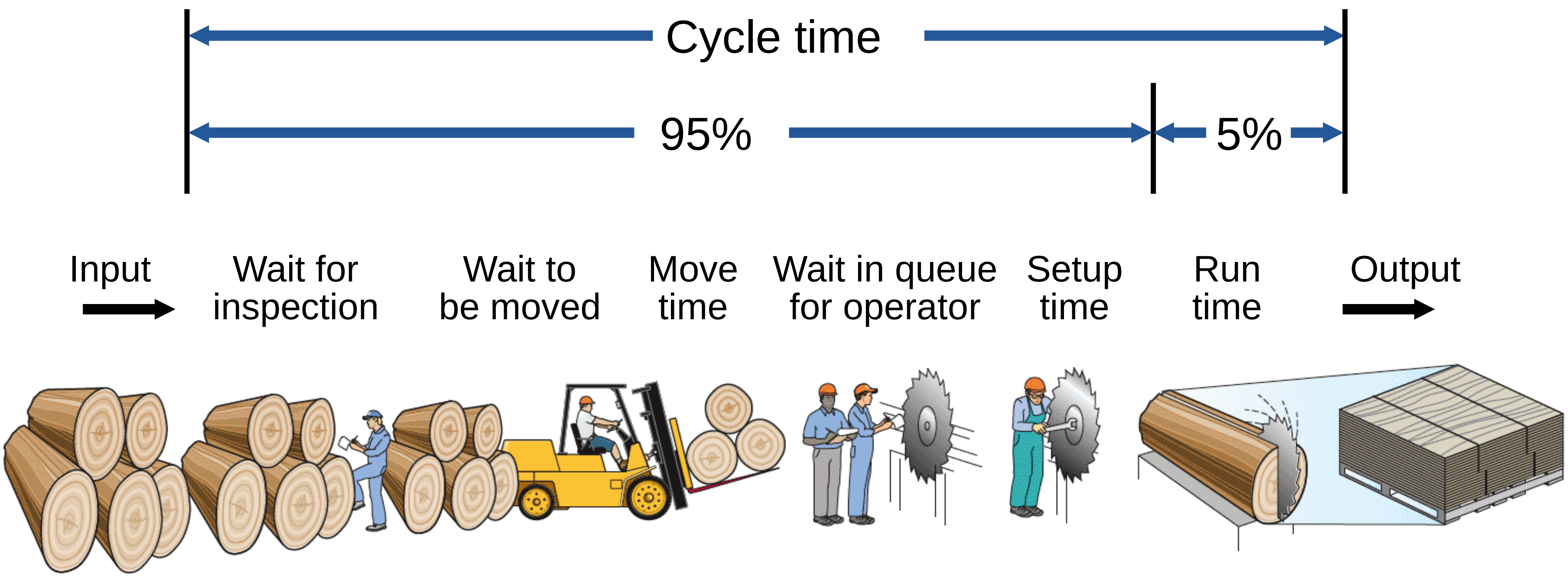
Raw Material
(Input)

Work In Process
(Transformation/Value Add)

Finished Goods
(Output)

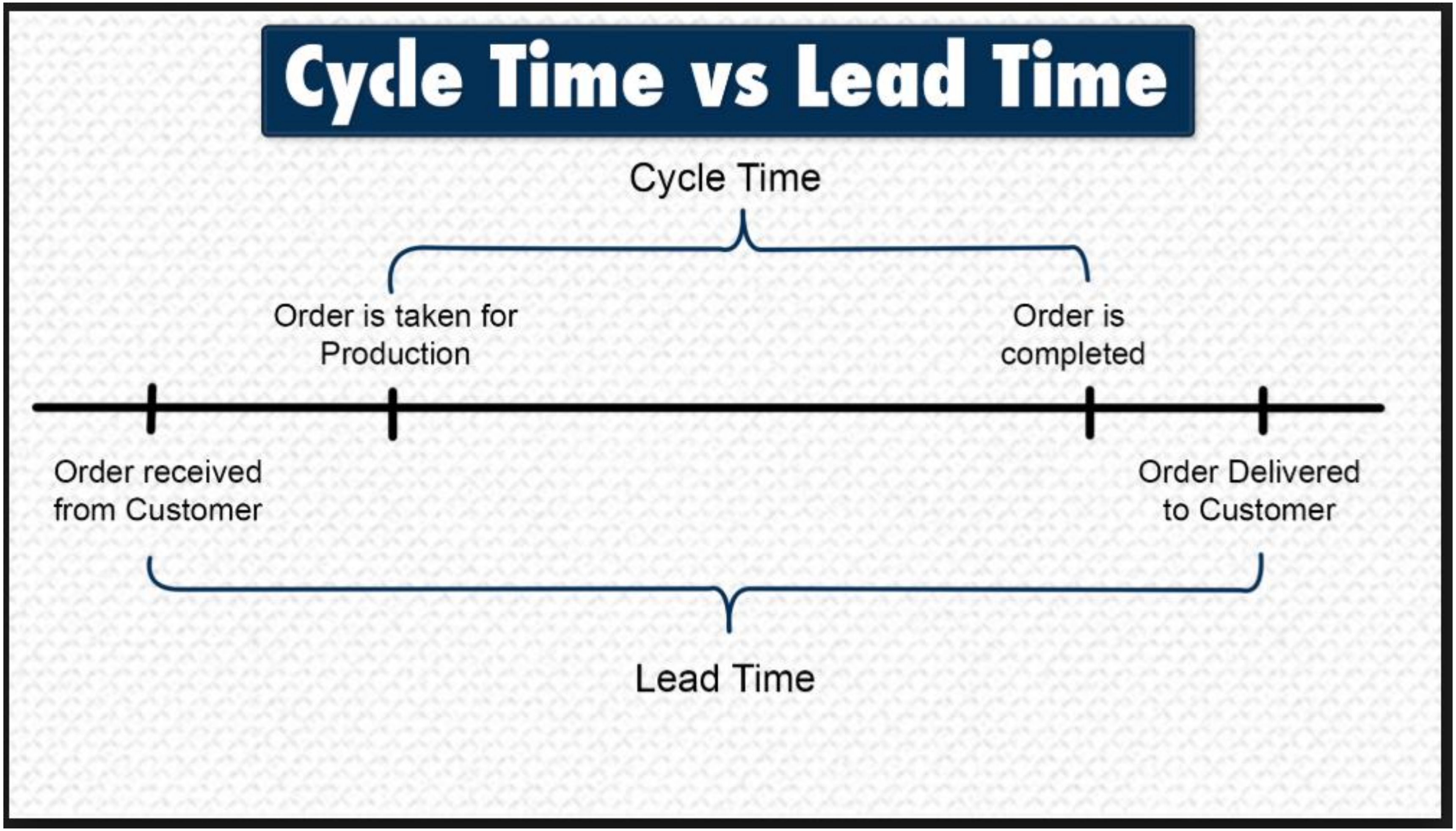
Cycle Time

- Most of the time that work is in-process (95% of the cycle time) is NOT productive time



Lead Time

Operations & Supply Chain University



Inventory Management Decisions

Operations & Supply Chain University

- Inventory managers deal with two fundamental decisions:
  1. *When* to order items from a supplier or when to initiate production runs if the firm makes its own items
  2. *How much* to order or produce each time a supplier or production order is placed

- Effective inventory management requires
  - A system to keep track of inventory
  - A reliable forecast of demand
  - Knowledge of lead times
  - Reasonable estimates of costs
  - A classification system (like ABC)

Inventory Characteristics

Operations & Supply Chain University

- Number of time periods in planning horizon: short or long planning horizon such as days, weeks, months, quarters, and years
- Size of time periods: hours, days, weeks, months, quarters
- Lead time: the time between placement of an order and its receipt
- Stockout: the inability to satisfy demand for an item
    - Backorder: when a customer is willing to wait for an item
    - Lost sale: when the customer is unwilling to wait and purchases the item elsewhere

Inventory Management Costs

Operations & Supply Chain University

- Stock-Keeping Unit (SKU): a single item or asset stored at a particular location
- Four major categories of inventory costs:
  1. Setup (or ordering) costs: the result of the work involved in placing purchase orders with suppliers or configuring tools, equipment, and machines within a factory to produce an item
  2. Holding (or carrying) costs: expenses associated with carrying inventory, such as interest, insurance, taxes, deterioration, etc.
  3. Unit cost (of the Stock-Keeping Units, SKUs): the price paid for purchased goods or the internal cost of producing them
  4. Shortage (or stockout) costs: the costs associated with a SKU being unavailable when needed to meet demand

Nature of Demand

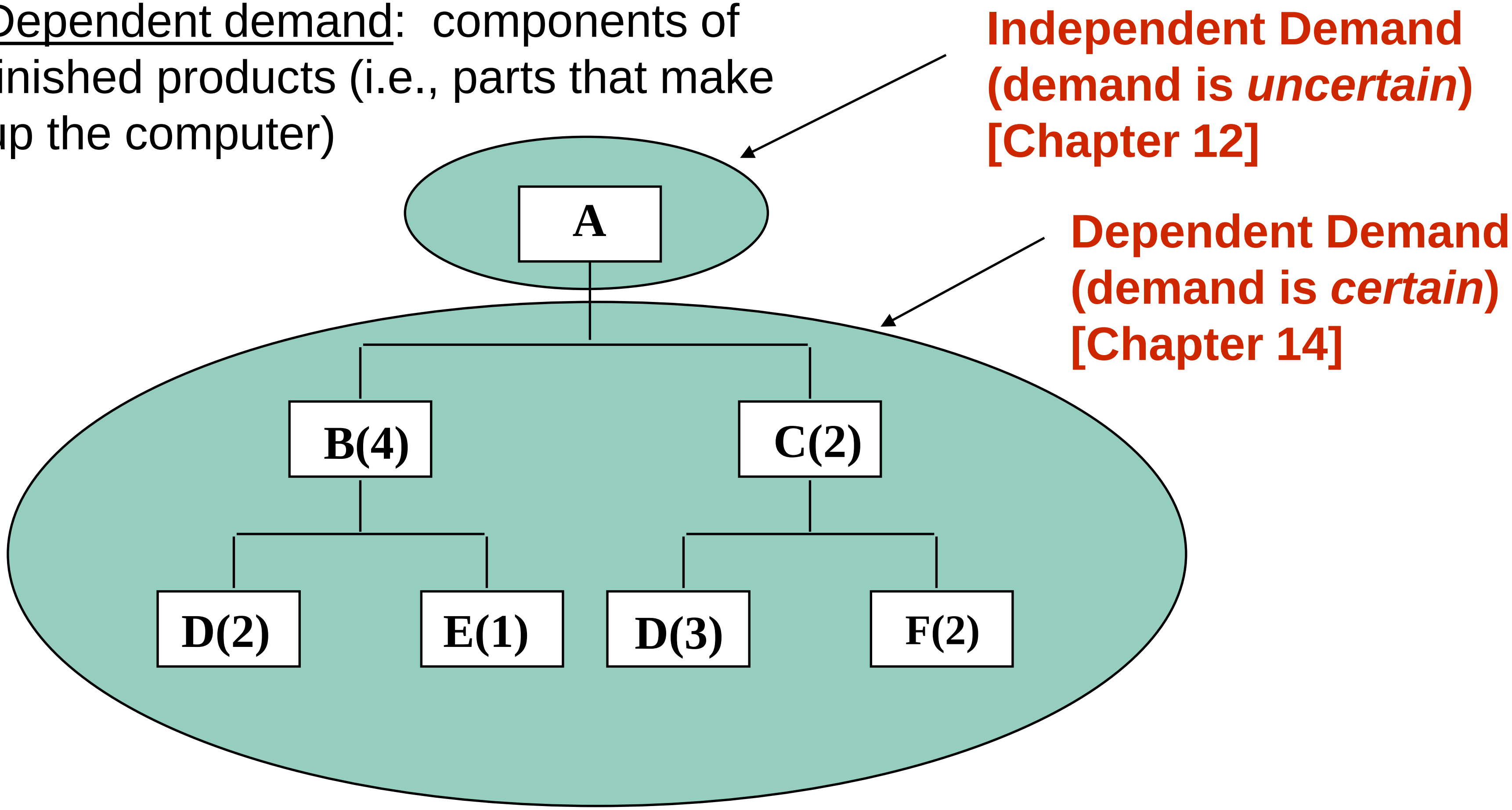
Operations & Supply Chain University

- Independent demand: demand for an SKU that is unrelated to the demand for other SKUs and needs to be forecast
- Dependent demand: demand directly related to the demand for other SKUs and can be calculated without needing to be forecast
- Static demand: stable demand
- Dynamic demand: varies over time

Inventory Models

Operations & Supply Chain University

- Independent demand: finished goods, items that are ready to be sold (i.e., a computer)
- Dependent demand: components of finished products (i.e., parts that make up the computer)



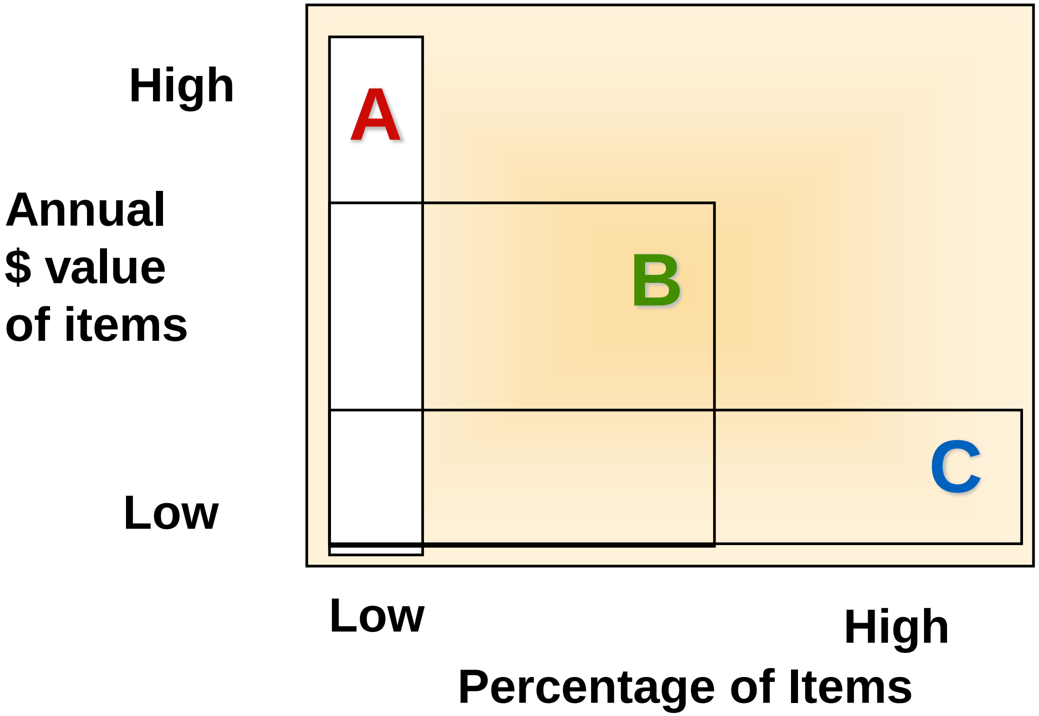
ABC Inventory Analysis

<u>ABC Analysis:</u> Classifying inventory according to some measure of importance and allocating control efforts accordingly (Pareto analysis: A vital few SKUs represent a high percentage of the total dollar inventory value)

A: very important

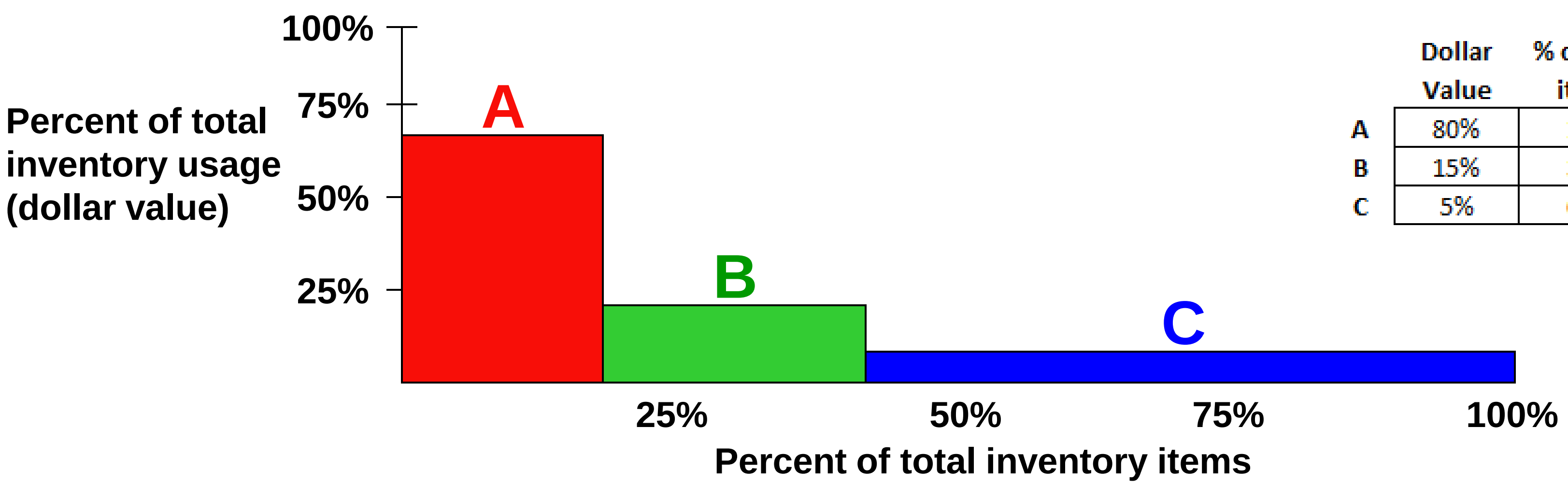
B: moderately important

C: least important



ABC Guidelines

- General guidelines:
  - **A** items account for a large dollar value but relatively small percentage of total items
    - 5% to 15% of items, yet 60% to **80%** of total dollar value
  - **B** items are medium dollar value (between A and C)
    - About 30% of items and **15%** dollar value
  - **C** items account for a small dollar value but a large percentage of total items
    - 50% to 60% of items, but only **5%** to 15% of dollar value



	Dollar Value	% of total items
A	80%	10%
B	15%	30%
C	5%	60%

ABC Analysis: Steps

Operations & Supply Chain University

1. Calculate annual dollar-value = Unit price x annual demand
2. Sort by decreasing order for annual dollar value (use Excel)
3. Calculate sum of total annual dollar $ value of all SKU's
4. Calculate % annual value per SKU
5. Calculate cumulative total $ value = Unit annual value + all previous
6. Calculate % annual cumulative value
7. Assign ABC codes per company policy

Example: ABC Analysis

SKU #	Unit Price	Annual Demand
#236-W	146.25	4,690
#236-Z	13.45	3,250
#456-M	9.35	7,200
#471-B	125.00	2,095
#479-J	3.50	9,000
#555-J	26.34	7,800
#635-P	99.50	6,970
#666-M	3.67	14,000
#765-N	56.95	4,960
#794-F	2.56	12,800
#851-H	79.50	890
#867-I	245.32	2,350
#993-X	34.65	7,800
#994-Y	35.65	7,200
		91,005

Example:

- Sample of 14 SKUs
- Total of ≈ 91,000 units

Example: ABC Analysis

Operations & Supply Chain University

SKU #	Unit Price	Annual Demand	Annual $ Value
#635-P	99.50	6,970	693,515
#236-W	146.25	4,690	685,913
#867-I	245.32	2,350	576,502
#765-N	56.95	4,960	282,472
#993-X	34.65	7,800	270,270
#471-B	125.00	2,095	261,875
#994-Y	35.65	7,200	256,680
#555-J	26.34	7,800	205,452
#851-H	79.50	890	70,755
#456-M	9.35	7,200	67,320
#666-M	3.67	14,000	51,380
#236-Z	13.45	3,250	43,713
#794-F	2.56	12,800	32,768
#479-J	3.50	9,000	31,500
		91,005	3,530,114

Annual Dollar Value =
Unit Price x Annual Demand
(Sorted in decreasing Annual Dollar Value)

i.e. #635P:
= $99.50 x 6970 units = $693,515

Example: ABC Analysis

Operations & Supply Chain University

SKU #	Unit Price	Annual Demand	Annual $ Value	% Annual Value
#635-P	99.50	6,970	693,515	20%
#236-W	146.25	4,690	685,913	19%
#867-I	245.32	2,350	576,502	16%
#765-N	56.95	4,960	282,472	8%
#993-X	34.65	7,800	270,270	8%
#471-B	125.00	2,095	261,875	7%
#994-Y	35.65	7,200	256,680	7%
#555-J	26.34	7,800	205,452	6%
#851-H	79.50	890	70,755	2%
#456-M	9.35	7,200	67,320	2%
#666-M	3.67	14,000	51,380	1%
#236-Z	13.45	3,250	43,713	1%
#794-F	2.56	12,800	32,768	1%
#479-J	3.50	9,000	31,500	1%
		91,005	3,530,114	

% Annual Dollar Value =
For each SKU:
Annual $ Value / Total Annual $
X 100%

i.e. #635P:
= $693,515 / $3,530,114 x 100%
= 0.20 x 100%
= 20%

Example: ABC Analysis

Operations & Supply Chain University

Cum $ = Sum of Annual $ Value

SKU #	Unit Price	Annual Demand	Annual $ Value	% Annual Value	Cum $
#635-P	99.50	6,970	693,515	20%	693,515
#236-W	146.25	4,690	685,913	19%	1,379,428
#867-I	245.32	2,350	576,502	16%	1,955,930
#765-N	56.95	4,960	282,472	8%	2,238,402
#993-X	34.65	7,800	270,270	8%	2,508,672
#471-B	125.00	2,095	261,875	7%	2,770,547
#994-Y	35.65	7,200	256,680	7%	3,027,227
#555-J	26.34	7,800	205,452	6%	3,232,679
#851-H	79.50	890	70,755	2%	3,303,434
#456-M	9.35	7,200	67,320	2%	3,370,754
#666-M	3.67	14,000	51,380	1%	3,422,134
#236-Z	13.45	3,250	43,713	1%	3,465,846
#794-F	2.56	12,800	32,768	1%	3,498,614
#479-J	3.50	9,000	31,500	1%	3,530,114
		91,005	3,530,114		

i.e., #236-W Cum $
= 693,515 + 685,913
= 1,379,428

i.e., #666-M Cum $
= 3,370,754 + 51,380
= 3,422,134

Example: ABC Analysis

Operations & Supply Chain University

% Annual = Cum $ / Total Annual $ x 100%

SKU #	Unit Price	Annual Demand	Annual $ Value	% Annual Value	Cum $	% Annual
#635-P	99.50	6,970	693,515	20%	693,515	20%
#236-W	146.25	4,690	685,913	19%	1,379,428	39%
#867-I	245.32	2,350	576,502	16%	1,955,930	55%
#765-N	56.95	4,960	282,472	8%	2,238,402	63%
#993-X	34.65	7,800	270,270	8%	2,508,672	71%
#471-B	125.00	2,095	261,875	7%	2,770,547	78%
#994-Y	35.65	7,200	256,680	7%	3,027,227	86%
#555-J	26.34	7,800	205,452	6%	3,232,679	92%
#851-H	79.50	890	70,755	2%	3,303,434	94%
#456-M	9.35	7,200	67,320	2%	3,370,754	95%
#666-M	3.67	14,000	51,380	1%	3,422,134	97%
#236-Z	13.45	3,250	43,713	1%	3,465,846	98%
#794-F	2.56	12,800	32,768	1%	3,498,614	99%
#479-J	3.50	9,000	31,500	1%	3,530,114	100%
		91,005	3,530,114			

i.e., #635-P
= 693,515 / 3,530,114
= 0.20 x 100%
= 20%

i.e., #456-M
=3,370,754/3,530,114
= 0.95 x 100%
= 95%

Example: ABC Analysis

Operations & Supply Chain University

SKU #	Unit Price	Annual Demand	Annual $ Value	% Annual Value	Cum $	% Annual
#635-P	99.50	6,970	693,515	20%	693,515	20%
#236-W	146.25	4,690	685,913	19%	1,379,428	39%
#867-I	245.32	2,350	576,502	16%	1,955,930	55%
#765-N	56.95	4,960	282,472	8%	2,238,402	63%
#993-X	34.65	7,800	270,270	8%	2,508,672	71%
#471-B	125.00	2,095	261,875	7%	2,770,547	78%
#994-Y	35.65	7,200	256,680	7%	3,027,227	86%
#555-J	26.34	7,800	205,452	6%	3,232,679	92%
#851-H	79.50	890	70,755	2%	3,303,434	94%
#456-M	9.35	7,200	67,320	2%	3,370,754	95%
#666-M	3.67	14,000	51,380	1%	3,422,134	97%
#236-Z	13.45	3,250	43,713	1%	3,465,846	98%
#794-F	2.56	12,800	32,768	1%	3,498,614	99%
#479-J	3.50	9,000	31,500	1%	3,530,114	100%
		91,005	3,530,114			

A items:
≈ 55% value

B items:
≈ 37% value

C items:
≈ 8% value

ABC Analysis

Operations & Supply Chain University

- Other criteria than annual dollar volume may be used
  - Anticipated engineering changes (“X” items)
  - Delivery problems
  - Quality problems
  - High unit cost

- Policies employed may include
  - More emphasis on supplier development for A items
  - Tighter physical inventory control for A items
  - More care in forecasting A items

Physical Inventories

Operations & Supply Chain University

- Physical inventory: the counting by hand, weight or bulk of all items in a company's inventory to validate record accuracy
  - Usually taken annually or semi-annually
- Cycle counting: the counting of inventory items on a continuous basis by an independent team of cycle counters. The counting activities are structured so each item is counted at least once a year
  - Advantages
    - Eliminates shutdowns and interruptions
    - Eliminates annual inventory adjustment
    - Trained personnel audit inventory accuracy
    - Allows causes of errors to be identified and corrected
    - Maintains accurate inventory records
  - Often used with ABC analysis to determine cycle

Cycle Counting Example

5,000 items in inventory, 500 A items, 1,750 B items, 2,750 C items.

Policy is to count A items every month (20 working days), B items every quarter (60 days), and C items every six months (120 days).

How many items will be counted in the cycle count process per day?

Item Class	Quantity	Cycle Counting Policy	Number of Items Counted per Day
A	500	Each month	500/20 = 25/day
B	1,750	Each quarter	1,750/60 = 29/day
C	2,750	Every 6 months	2,750/120 = 23/day
			77/day

Record Accuracy

Operations & Supply Chain University

- Record accuracy
  - Accurate records are a critical ingredient in production and inventory systems
  - Allows organization to focus on what is needed
  - Necessary to make precise decisions about ordering, scheduling, and shipping
  - Incoming and outgoing record keeping must be accurate
  - Stockrooms should be secure

- Inventory shrinkage (or inventory shrink): the loss of products between point of manufacture or purchase from supplier and point of sale
  - Shrinkage = Booked Inventory - Physical Counted Inventory

Inventory Shrinkage

Operations & Supply Chain University

- Shrinkage is approximately 1.44% of retail sales

Source of Inventory Shrinkage	% of Loss*	$ Lost
Employee Theft	43%	$14.4 billion
Shoplifting	35%	$11.7 billion
Administrative Error	14.5%	$4.9 billion
Vendor Fraud	3.8%	$1.3 billion
Total Inventory Shrinkage		$33.5 billion

*total not equal to 100% due to rounding

- Shrinkage is *decreasing* on a yearly basis due to technology

Source: National Retail Security Survey

Cycle Counting & Order Policy Example

Operations & Supply Chain University

Purchasing Guidelines - Pareto Analysis

ABC	%	PART COUNT	ANNUAL SPEND	Annual Usage Threshold ($K)	Annual Usage $ average	Annual Deliveries	Average Delivery dollar value ($)	Strategic Plan	Typical Purchase Order structure	Cycle Count Covered Policy
A	**80**	**1,274**	**$26.1M**	$4,400+	$20,461	12 - 52	$950	Kanban, Consignment, EOQ, ROP	Blanket PO + Vendor Stocking Agreement	**A Code =** High dollar = Cycle Count BI-MONTHLY (2 months)
B	**15**	**2,387**	**$4.9M**	$900 - $4,399	$2,078	2 - 4	$700	Schedule quarterly deliveries	Blanket PO	**B Code** = Medium dollar = Cycle Count TWICE per year (6 months)
C	**5**	**8,931**	**$1.6M**	$0 - $899	$183	1 - 2	$500	Large qty deliveries, don't be late	Regular PO	**C Code** = Low dollar = Cycle Count ONCE per year (12 months)
	100	**12,592**	**$32.6M**							

ADDITIONAL COMMENTS:

For long lead time "C" items (12+ weeks), purchase the full EAU and bring in as soon as parts are available

Do not schedule line items less than $100. Bring them in if they are available. If not available, schedule per part LT (not MRP need date)

Inventory Models for Independent Demand

- Three independent demand models
  1. Basic Economic Order Quantity (EOQ) model
  2. Economic Production Quantity (EPQ) model
  3. Quantity Discount model

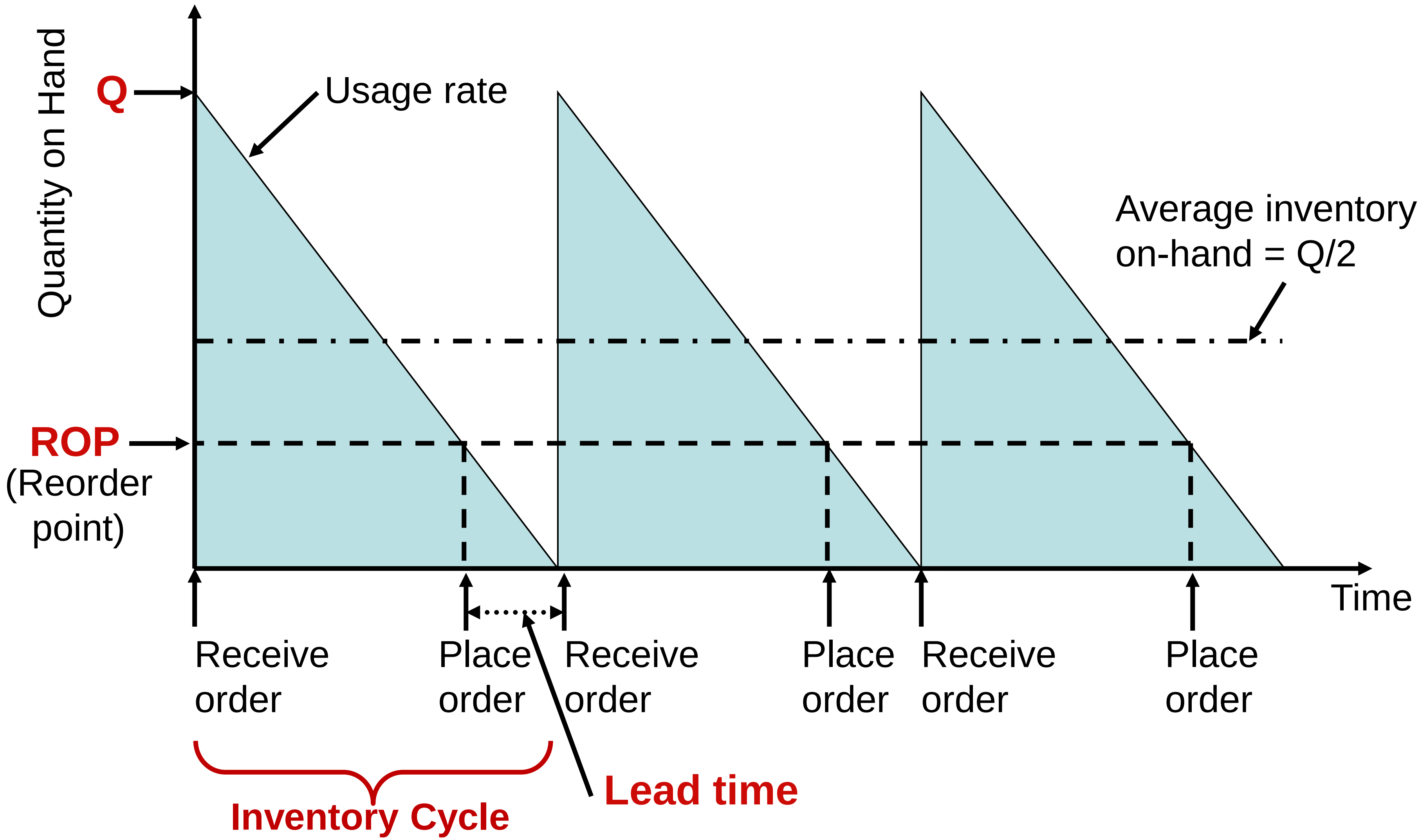
Economic Order Quantity Model

Operations & Supply Chain University

- Economic Order Quantity (EOQ) model: A classic model developed to minimize total annual inventory cost

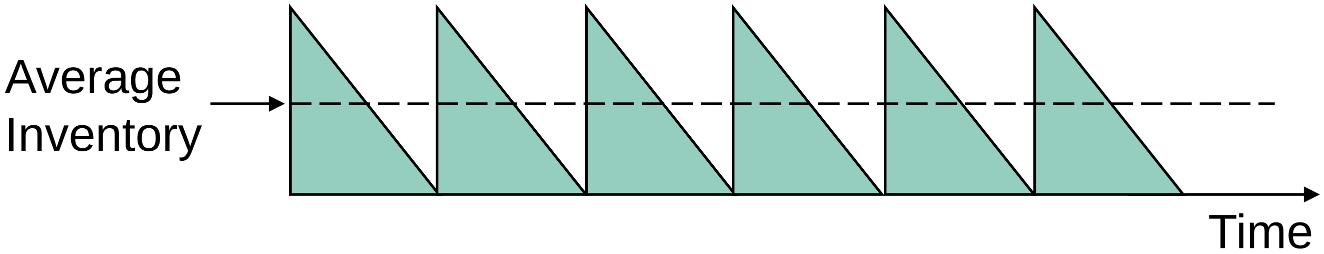
- EOQ assumptions:
  - Only one product is involved (one SKU)
  - Demand is known, constant and independent of decisions for other items
  - The entire order quantity (Q) is received in a single delivery
  - Lead time (time between placement and receipt of an order) is known and constant
  - Quantity discounts are not possible
  - Only two types of costs are relevant – setup/order costs and holding/carrying costs
  - Stockouts (shortages) can be completely avoided if orders are placed at the right time

The Inventory Cycle

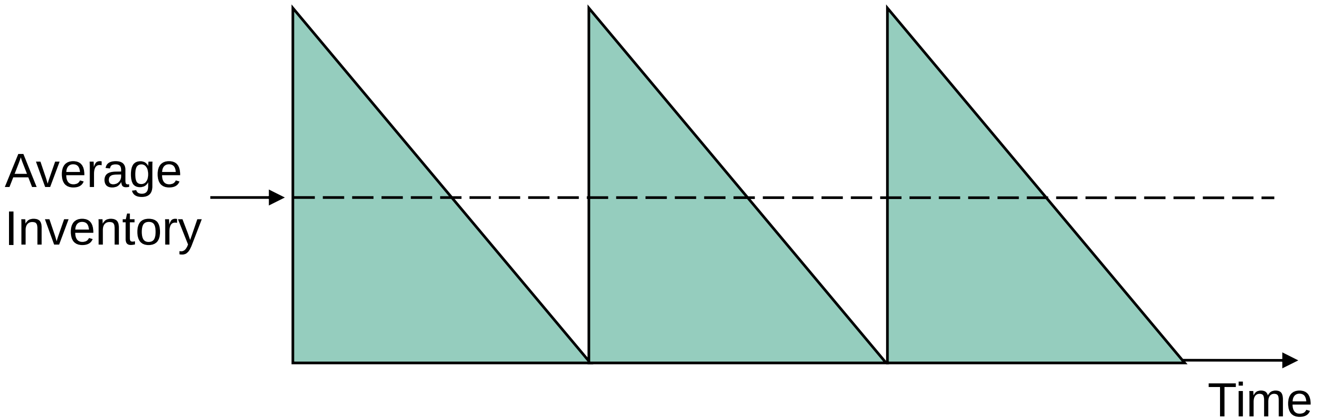


Average Inventory Levels and Numbers of Orders

Operations & Supply Chain University



Many orders produce a low average inventory



Few orders produce a high average inventory

Economic Order Quantity

Operations & Supply Chain University

- The Economic Order Quantity (EOQ) model is a classic economic model developed in the early 1900's that **minimizes** total cost, which is the sum of the inventory-holding cost (H) and the set-up/ordering cost (S)

  Q = number of units per order

  Q* = optimum number of units per order = EOQ

  D = annual demand

$$\text{Annual inventory } \textbf{holding} \text{ cost} = \left(\text{average inventory}\right)\left(\text{annual holding cost per unit}\right) = \frac{Q}{2} H$$

$$\text{Annual } \textbf{ordering} \text{ cost} = \left(\text{number of orders per year}\right)\left(\text{setup cost per order}\right) = \left(\frac{D}{Q}\right) S$$

Total Annual Cost

- Total Annual Cost (TC) = annual set-up cost + annual holding cost:

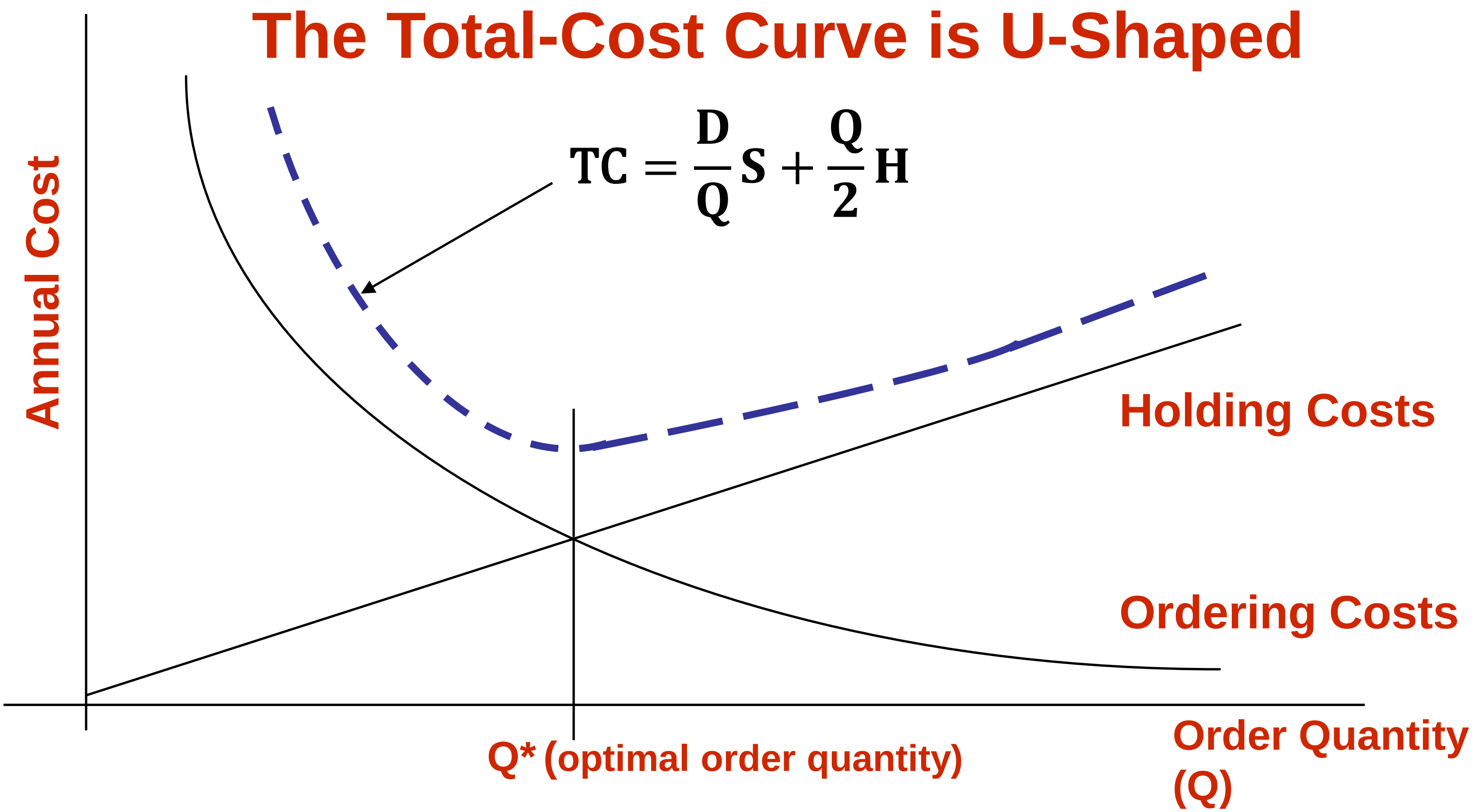
$$TC = \frac{D}{Q}S + \frac{Q}{2}H$$

- Optimal Order Quantity (Q*): order quantity that minimizes the total cost expressed in the equation above
- Q* is also known as the Economic Order Quantity, or EOQ:

$$Q* = \sqrt{\frac{2DS}{H}}$$

- N = Orders per year: D/Q
  - D = Annual Demand
  - Q = Number of units per order
- T = Time between orders: # working days/N

Total Cost Curve



EOQ Example #1

A local distributor for a national tire company expects to sell approximately 9,600 steel-belted radial tires of a certain size and tread design next year. Annual carrying cost is $16 per tire, and ordering cost is $75. The distributor operates 288 days a year.

a. What is the EOQ?
b. How many orders per year are expected?
c. What is the expected time between orders?
d. What is the total annual cost if the EOQ quantity is ordered?

EOQ Example #1 - Solution

Demand = D = 9600 tires/year

Holding cost = H = $16 per tire/year

Setup cost = S = $75 /order, 288 working days/year

$$Q* = \sqrt{\frac{2DS}{H}}$$

$$TC = \frac{D}{Q}S + \frac{Q}{2}H$$

a. $$\text{EOQ} = Q* = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(9600)(75)}{16}} = \textbf{300 tires}$$

b. $$N = \text{Orders per year} = \frac{D}{Q*} = \frac{9600}{300} = \textbf{32 orders}$$

c. $$T = \text{Expected time between orders} = \frac{\text{\# working days}}{N} = \frac{\text{288 days}}{\text{32 orders}} = \textbf{9 days}$$

d. $$TC = \text{Setup Costs} + \text{Holding Costs}$$

$$TC = \frac{D}{Q*}S + \frac{Q*}{2}H = \frac{9600}{300}(75) + \frac{300}{2}(16) = \$2400 + \$2400 = \$\mathbf{4800}$$

So, if using Q*, setup cost = holding cost, and Total Cost is minimum

However, we might choose to use a different Q. Let's order 250 tires instead:

$$TC = \frac{D}{Q}S + \frac{Q}{2}H = \frac{9600}{250}(75) + \frac{250}{2}(16) = \$2880 + \$2000 = \$\mathbf{4880}$$

Setup cost increases (more orders), holding cost decreases (fewer per order)

EOQ Example #2

Operations & Supply Chain University

Watchem Manufacturing assembles security monitors. It purchases 3,600 black and white Cathode Ray Tubes (CRTs) a year at $65 each. Ordering costs are $31, and annual carrying costs are 20 percent of the purchase price. Watchem builds products 252 days per year. Compute the optimal quantity and the total annual cost of ordering and carrying the inventory?

a. What is the EOQ?
b. How many orders per year are expected?
c. What is the expected time between orders?
d. What is the total annual cost if the EOQ quantity is ordered?

EOQ Example #2 - Solution

Operations & Supply Chain University

Demand = D = 3600 tubes/year
Price = $65/unit
Holding cost = H = 20%Price
Setup cost = S = $31 /order, 252 working days per year

$$Q* = \sqrt{\frac{2DS}{H}} \qquad TC = \frac{D}{Q}S + \frac{Q}{2}H$$

Holding cost = H = 20%Price → H=(0.20)($65/unit) = $13/unit

a. $EOQ = Q* = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(3600)(31)}{13}} = \textbf{131 CRTs}$

b. $N = \text{Orders per year} = \frac{D}{Q*} = \frac{3600}{131} = \textbf{27.4 = 28 orders}$

c. $T = \text{Expected time between orders} = \frac{\text{\# working days}}{N} = \frac{252 \text{ days}}{28 \text{ orders}} = \textbf{9 days}$

d. $TC = \text{Setup Costs} + \text{Holding Costs}$

$$TC = \frac{D}{Q*}S + \frac{Q*}{2}H = \frac{3600}{131}(31) + \frac{131}{2}(13) = \$852 + \$852 = \$\mathbf{1704}$$

So, if using $Q*$, setup cost = holding cost, and Total Cost is minimum

When to Reorder?

- ReOrder Point (ROP): When the quantity on hand of an item drops to this amount, the item is reordered
- Determinants of the ReOrder Point (ROP)
  - The rate of demand
  - The lead time
  - Demand and/or lead time variability
  - Stockout risk (safety stock)
- (For this class, we will assume constant demand and lead time): Reorder point:

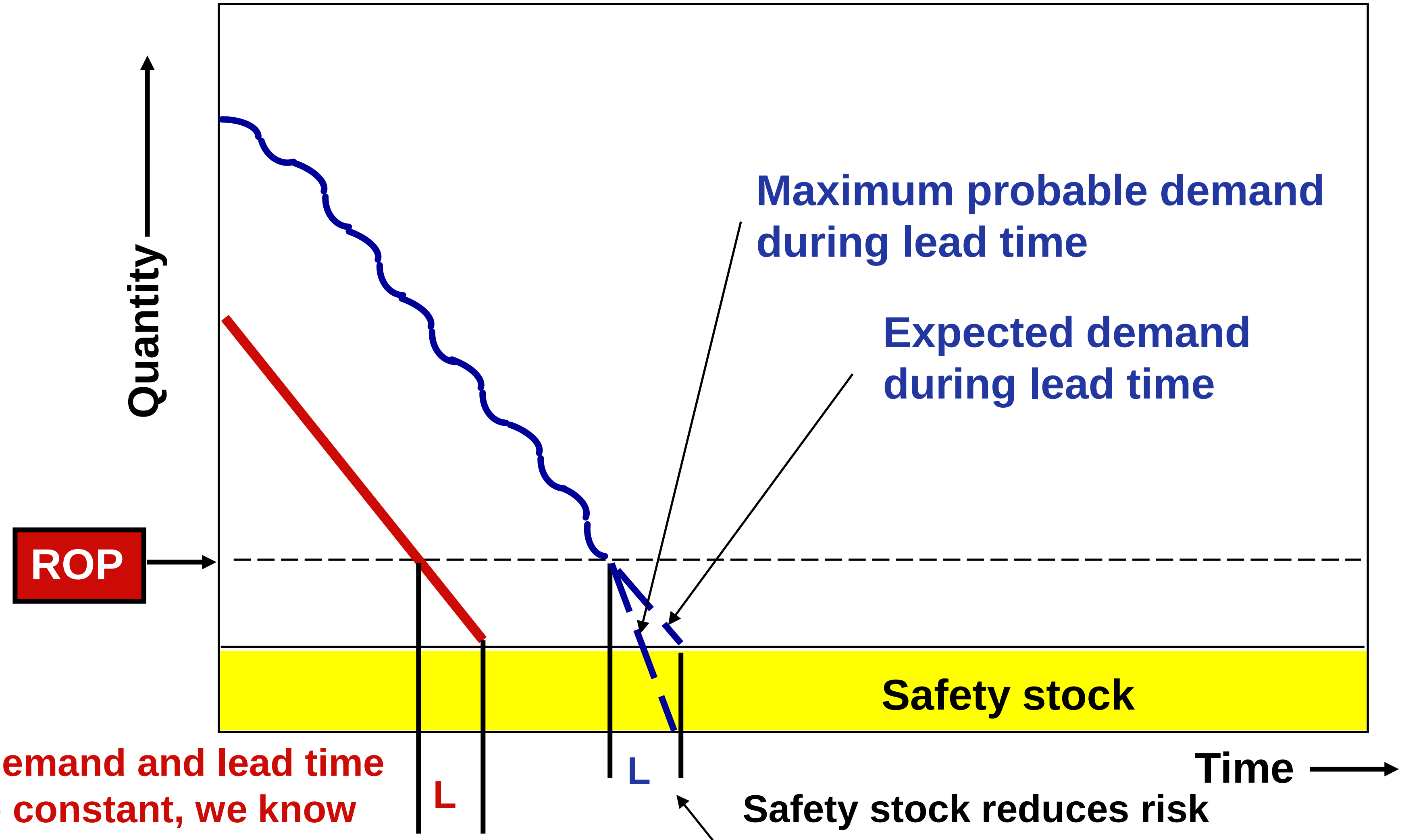
  ROP = d x L

  ROP = (demand per day) x (lead time for a new order)

  d = D/number of working days per year
- Safety Stock: Stock that is held in excess of expected demand due to variable demand rate and/or lead time

Safety Stock

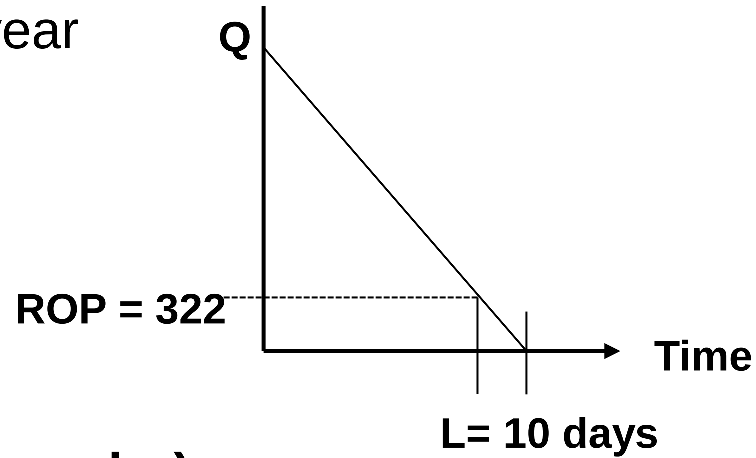
Operations & Supply Chain University



If demand and lead time are constant, we know exactly when to reorder

Reorder Point (ROP) Example

- Given:
  - Demand = D = 10,000 silicon chips/year
  - Store open 311 days/year
  - Lead time = L = 10 days
- When do we reorder?



Reorder point: ROP = d x L
= (demand per day) x (lead time for a new order)

Daily demand = d
= (10,000 chips/yr) / (311 days per yr) = 32.2 chips/day
ROP = d(L) = (32.2 chips/day)(10 days) = 322 chips

Inventory Models for Independent Demand

Operations & Supply Chain University

- Three independent demand models
  1. Basic Economic Order Quantity (EOQ) model
  2. Economic Production Quantity (EPQ) model
     - The textbook calls this the Production Order Quantity (POQ) model
  3. Quantity Discount model

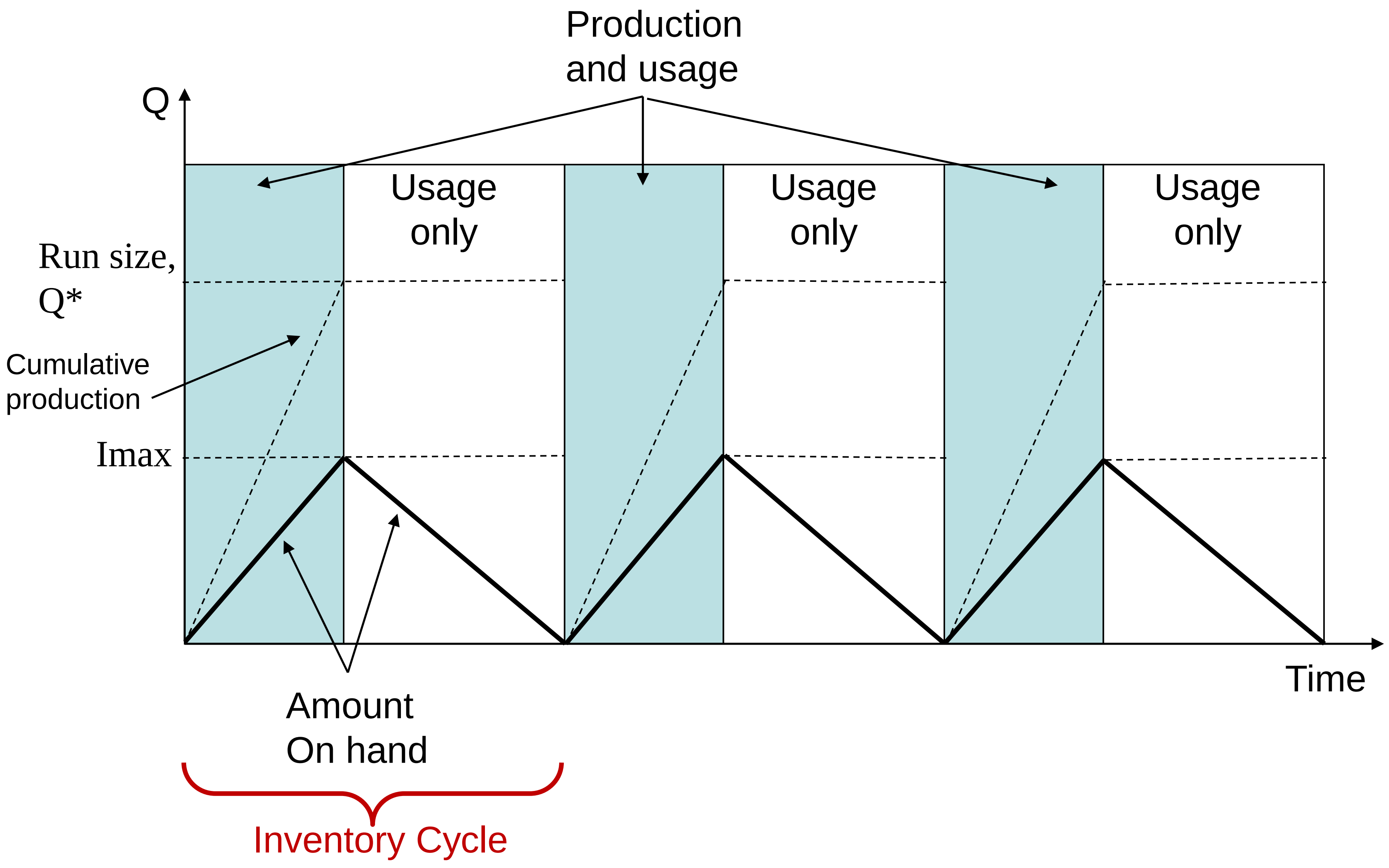
Economic Production Quantity (EPQ)

Operations & Supply Chain University

- Economic Production Quantity (EPQ): Used when inventory builds up over a period of time after an order is placed.

- Used when units are produced and sold simultaneously
- Assumptions of EPQ are similar to EOQ except orders are received incrementally during production
  - Only one item is involved (one SKU)
  - Annual demand is known
  - Usage rate is constant
  - Usage occurs continually but production occurs periodically
  - Production rate is constant
  - Lead time is constant
  - No quantity discounts

EPQ with Incremental Inventory Replenishment



Economic Production Quantity (EPQ) Equations

Operations & Supply Chain University

The production run quantity Q_P* is:

$$Q_P^* = \sqrt{\frac{2DS}{H[1-(d/p)]}}$$

POQ formula

Where:

p = daily production or *delivery* rate

d = daily demand or *usage* rate

Alternate equation (makes more sense intuitively and easier to calculate):

EOQ formula

$$Q_P^* = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-d}}$$

EPQ formula

Economic Production Quantity (EPQ) Equations

Operations & Supply Chain University

Total Cost: $TC = \frac{D}{Q}S + \frac{I\text{-}max}{2}H$

Maximum Inventory level: $I\text{-}max = Q\left(1 - \frac{d}{p}\right)$

Number of production runs: $N = \frac{D}{Q}$

Expected time between orders: $\frac{\text{Number of working days/year}}{N}$

$$Q_P{}^{*} = \sqrt{\frac{2DS}{H[1-(d/p)]}} \quad \text{or} \quad Q_P{}^{*} = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-d}}$$

Economic Production Quantity (EPQ) Example #1

Operations & Supply Chain University

Nathan Manufacturing makes and sells specialty hubcaps for the retail automobile aftermarket. Nathan's forecast for its wire-wheel hubcap is 1000 units next year, with an average daily demand of 4 units. However, the production process is most efficient at 8 units per day. So the company produces 8 per day but uses only 4 per day. The company wants to solve for the optimum number of units per order. Setup costs are $10 and Holding costs are $.50 per unit. Note: this plant schedules production of the hubcap only as needed, during the 250 days per year the shop operates.

Annual demand = D = 1000 units
Setup costs = S = $10
Holding Costs = H = $0.50 per unit per year
Daily production rate = p = 8 units/day
Daily demand rate = d = 4 units/day
(Note: use of 4 units/day given, but could calculate:
demand = 1000 units per year/250 days per year = 4 units/day)

Economic Production Quantity (EPQ) Solution

Operations & Supply Chain University

POQ Equation in text:

$$Q_P^* = \sqrt{\frac{2DS}{H[1-(d/p)]}}$$

EPQ Alternate equation:

$$Q_P^* = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-d}}$$

$$Q_P^* = \sqrt{\frac{2(1000)(10)}{0.50[1-(4/8)]}}$$

$$= \sqrt{\frac{20{,}000}{0.50(1/2)}} = \sqrt{80{,}000}$$

$$= 282.8 = \textbf{283} \text{ hubcaps}$$

$$Q_P^* = \sqrt{\frac{2(1000)(10)}{0.50}}\sqrt{\frac{8}{8-4}}$$

$$= \sqrt{\frac{20{,}000}{0.50}}\sqrt{2} = (200)(1.41)$$

$$= 282.8 = \textbf{283} \text{ hubcaps}$$

Either equation provides same result!

Economic Production Quantity (EPQ) Example #2

Operations & Supply Chain University

A toy manufacturer uses 48,000 rubber wheels per year for its popular dump truck series. The firm makes its own wheels, which it can produce at a rate of 800 per day. The toy trucks are assembled uniformly over the entire year. Carrying cost is $1 per wheel a year. Setup cost for a production run of wheels is $45. The firm operates 240 days per year. Determine the:

a. Optimal run size (EPQ)
b. Maximum inventory level (I-max)
c. Minimum total cost for carrying and setup (TC)

EPQ Solution

Operations & Supply Chain University

Demand = D = 48000 wheels per year		
Setup Cost = S = $45 per order		
Holding Cost = H = $1 per wheel/year		
Production rate = p = 800 wheel per day		
Daily demand = d = 48000 wh/year / 240 day/yr = 200 wh/day		

a. $Q_P{}^* = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-d}}$

$$Q_P{}^* = \sqrt{\frac{2(48000)(45)}{1}}\sqrt{\frac{800}{800-200}} = \mathbf{2,400\ EPQ}$$

$$Q*_P = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-d}}$$

$$\text{Maximim Inventory level} = I_{max} = Q\left(1-\frac{d}{p}\right)$$

b. $I_{max} = (2400)\left(1-\frac{200}{800}\right) = \mathbf{1,800\ wheels}$

$$TC = \frac{D}{Q}S + \frac{I-max}{2}H$$

c. $TC = \frac{D}{Q}S + \frac{I-max}{2}H = \frac{48000}{2400}45 + \frac{1800}{2}(1) = 900 + 900 = \$\mathbf{1,800}$

Inventory Models for Independent Demand

Operations & Supply Chain University

- Three independent demand models
  1. Basic Economic Order Quantity (EOQ) model
  2. Economic Production Quantity (EPQ) model
  3. Quantity Discount model

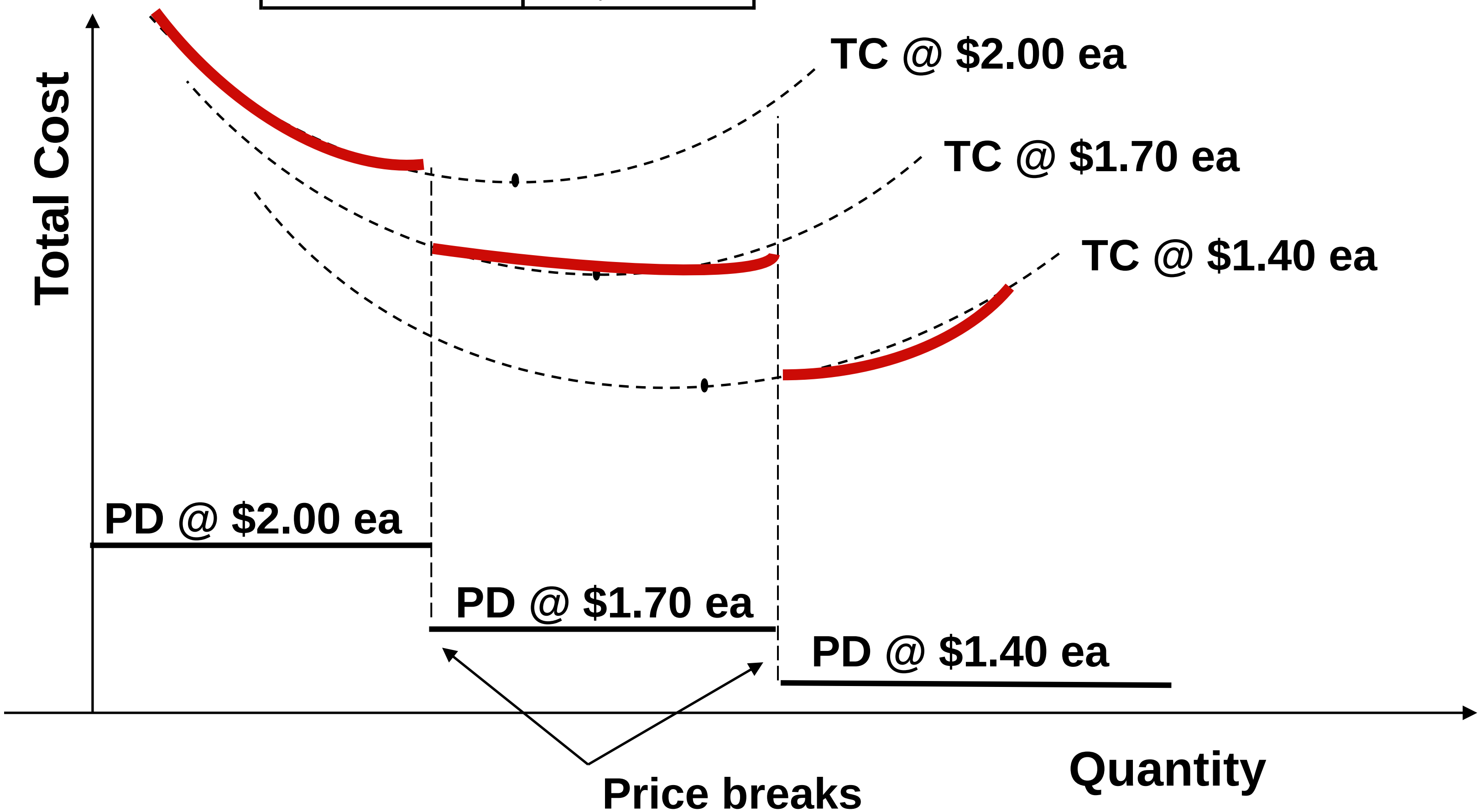
Quantity Discount Model

- Many suppliers offer discounts for purchasing larger quantities of goods because of economies of scale of shipping larger loads, not having to break apart boxes of items, or an incentive to increase total revenue
- To incorporate into the basic EOQ model, you must include purchase cost of item in total cost equation
- Purchase cost = Unit Price (P) x Annual Demand (D)

$$TC = \frac{D}{Q}S + \frac{Q}{2}H + \mathbf{PD}$$

Quantity Discount Model

Order Quantity	Price per Box
1 to 44	$2.00
45 - 69	$1.70
70 or more	$1.40



Two Models: Price Breaks

Operations & Supply Chain University

1. **Holding Costs are constant**
   - **Compute EOQ**
   - **Compute the Total Cost for all price breaks with lower unit cost (use smallest quantity in price break)**
2. Holding Costs as a % of price
   - Starting with lowest unit price, compute EOQ
   - If EOQ in lowest price range – finished
   - If not, then compute total costs for next lower price ranges

Quantity Discount Example #1

Operations & Supply Chain University

The maintenance department of a large hospital uses about 816 cases of liquid cleanser annually. Ordering costs are $12, carrying costs are $4 per case a year, and the new price schedule indicates that orders of less than 50 cases will cost $20 per case, 50-79 cases will costs $18 per case, 80-99 cases will cost $17 per case, and the larger orders will cost $16 per case. Determine the optimal order quantity and the total cost.

Quantity Discount Example #1 Solution

Operations & Supply Chain University

$$TC = \frac{D}{Q}S + \frac{Q}{2}H + PD$$

Demand = D = 816 cases per year
Setup Cost = S = $12 per order
Holding Cost = H = $4 per case per year

Quantity	Price
1 - 49	$20
50 - 79	18
80 - 99	17
100+	16

$$EOQ = Q* = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(816)(12)}{4}} = \mathbf{70}\text{ cases}$$

$$TC = \text{Setup Costs} + \text{Holding Costs} + PD$$

$$TC_{70} = \frac{D}{Q*}S + \frac{Q*}{2}H + PD = \frac{816}{70}(12) + \frac{70}{2}(4) + (18)816$$

$$TC_{70} = \$140 + 140 + 14{,}688 = \$\mathbf{14,968}$$

But we see there is a lower price if we buy more than the EOQ!

$$TC_{80} = \frac{D}{Q}S + \frac{Q}{2}H + PD = \frac{816}{80}(12) + \frac{80}{2}(4) + (17)816 = \mathbf{\$14,154}$$

But we see there is another lower price, so check it!

$$TC_{100} = \frac{D}{Q}S + \frac{Q}{2}H + PD = \frac{816}{100}(12) + \frac{100}{2}(4) + (16)816 = \mathbf{\$13,354}$$

So, for THIS EXAMPLE, TC for 100 cases is lowest and best choice

Two Models: Price Breaks

Operations & Supply Chain University

1. Holding Costs are constant
    - Compute EOQ
    - Compute the Total Cost for all price breaks with lower unit cost (use smallest quantity in price break)
2. **Holding Costs as a % of price**
    - **Starting with *lowest* unit price, compute EOQ**
    - **If EOQ in lowest price range – *finished***
    - **If not, then compute total costs for next lower price ranges**

Quantity Discount Example #2

Operations & Supply Chain University

Surge Electric uses 4,000 toggle switches a year. Switches are priced as follows: 1 to 499, 90 cents each; 500-999, 85 cents each; and 1,000 or more, 80 cents each. It costs approximately $30 to prepare an order and receive it, and carrying costs are 40 percent of purchase price per unit on an annual basis. Determine the optimal order quantity and the total annual cost

Quantity Discount Example #2 Solution

Operations & Supply Chain University

$$TC = \frac{D}{Q}S + \frac{Q}{2}H + PD$$

Demand = D = 4000 switches per year	
Setup Cost = S = $30 per order	
Holding Cost = H = 40% Price = 0.40P	

Quantity	Price	H = 0.4P
1 - 499	$0.90	= 0.4(0.90) = 0.36
500 - 999	$0.85	= 0.4(0.85) = 0.34
1000+	$0.80	= 0.4(0.80) = 0.32

Start with lowest price and check if Q* is "feasible":

$$EOQ = Q^*_{(0.80)} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(4000)(30)}{0.32}} = 866 \text{ switches}$$

Cost of $0.80 only valid for quantity of 1000 + switches, <u>NOT</u> FEASIBLE!

Check next lowest price and see if Q* is "feasible":

$$EOQ = Q^*_{(0.85)} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(4000)(30)}{0.34}} = 840 \text{ switches}$$

Because the quantity of 840 is in the (500-999) range, it <u>IS FEASIBLE</u>!

There is only one feasible quantity (but check Q* for $0.90 to verify)

$$EOQ = Q^*_{(0.90)} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(4000)(30)}{0.36}} = 816 \text{ switches}$$

Cost of $0.90 is only for quantities 1-499, <u>NOT</u> FEASIBLE!

Quantity Discount Example #2 Solution

Operations & Supply Chain University

$$TC = \frac{D}{Q}S + \frac{Q}{2}H + PD$$

Demand = D = 4000 switches per year	
Setup Cost = S = $30 per order	
Holding Cost = H = 40% Price = 0.40P	

Quantity	Price	H = 0.4P
1 - 499	$0.90	= 0.4(0.90) = 0.36
500 - 999	$0.85	= 0.4(0.85) = 0.34
1000+	$0.80	= 0.4(0.80) = 0.32

$$EOQ = Q^*_{(0.85)} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(4000)(30)}{0.34}} = 840 \text{ switches}$$

Because a quantity of 840 is in the (500-999) range, it IS FEASIBLE!
So, calculate Total Cost using quantity of 840

$$TC_{840} = \frac{D}{Q^*}S + \frac{Q^*}{2}H + PD = \frac{4000}{840}(30) + \frac{840}{2}(0.34) + (0.85)4000 = \$3686$$

But there is a lower price, so need to see if it is better

$$TC_{1000} = \frac{D}{Q}S + \frac{Q}{2}H + PD = \frac{4000}{1000}(30) + \frac{1000}{2}(0.32) + (0.80)4000 = \$3480$$

So, for THIS EXAMPLE, the minimum cost order size is 1000 switches

Final Words on Quantity Discounts

Operations & Supply Chain University

- The lowest price does not always yield the lowest Total Cost because Holding Costs are increasing
- Be sure to test combinations until you find the lowest Total Cost
  - Once Q* is calculated, calculate Total Cost at that price
  - If there is a lower price, calculate Total Cost again
  - Repeat process until you run out of lower prices OR when the Total Cost is higher than that at a higher price