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Chapter 6S
Statistical Process Control

Chapter 6S Learning Outcomes:

- Explain the purpose and use of a control chart
- Build x-charts and R-charts
- List the steps involved in building control charts
- Build p-charts and c-charts
- Explain process capability and compute C_p and C_{pk}

Quality Control Systems

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- Quality Control (QC): to ensure that a good or service conforms to specifications and meets customer requirements by monitoring and measuring processes and making any necessary adjustments to maintain a specified level of performance

- Quality Control Systems have three components:
  - A performance standard or goal
  - A means of measuring actual performance
  - Comparison of actual performance with the standard to form the basis for corrective action

- Quality at the source: the people responsible for the work control the quality of their processes by identifying and correcting any defects or errors when they first are recognized or occur

Importance of a Good Control System: 1:10:100 Rule

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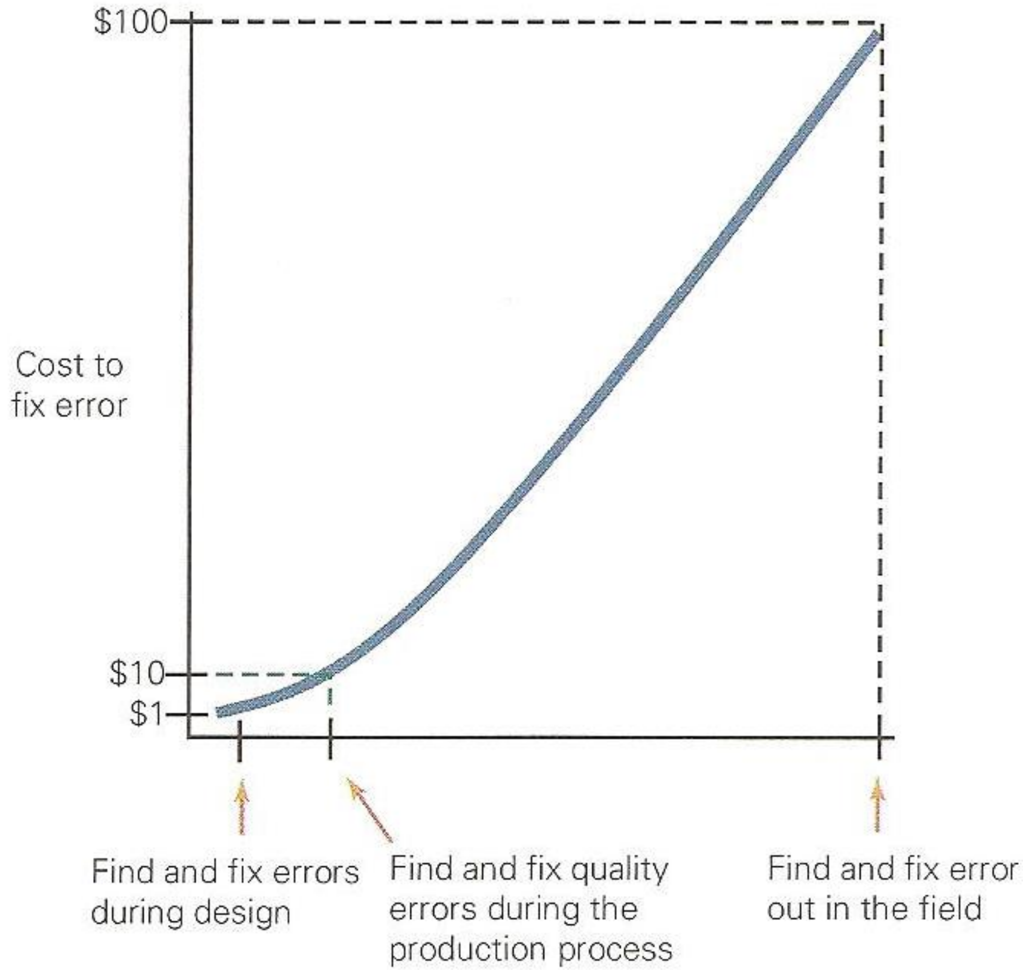

Quality Control (QC) Practices

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<u>QC Practices in Manufacturing:</u>

- Supplier Certification and Management: ensures conformance to requirements before value-adding operations begin
- In-process control: ensures that defective outputs do not leave the process and prevents defects in the first place
- Finished goods control: verifies that product meets customer requirements

<u>QC Practices in Services:</u>

- Prevent sources of errors and mistakes in the first place by using poka-yoke approaches
- Customer satisfaction measurement with actionable results (responses that are tied directly to key business processes)
- Many quality control tools and practices apply to both goods and services

Statistical Process Control

- Statistical Process Control (SPC): a methodology for monitoring quality of manufacturing and service delivery processes to help identify and eliminate unwanted causes of variation
  - The essence of statistical process control is to assure that the output of a process is random so that future output will be random
- Four key steps
  1. Measure the process
  2. When a change is indicated, find the assignable cause
  3. Eliminate or incorporate the cause
  4. Restart the revised process

Process Control Procedures

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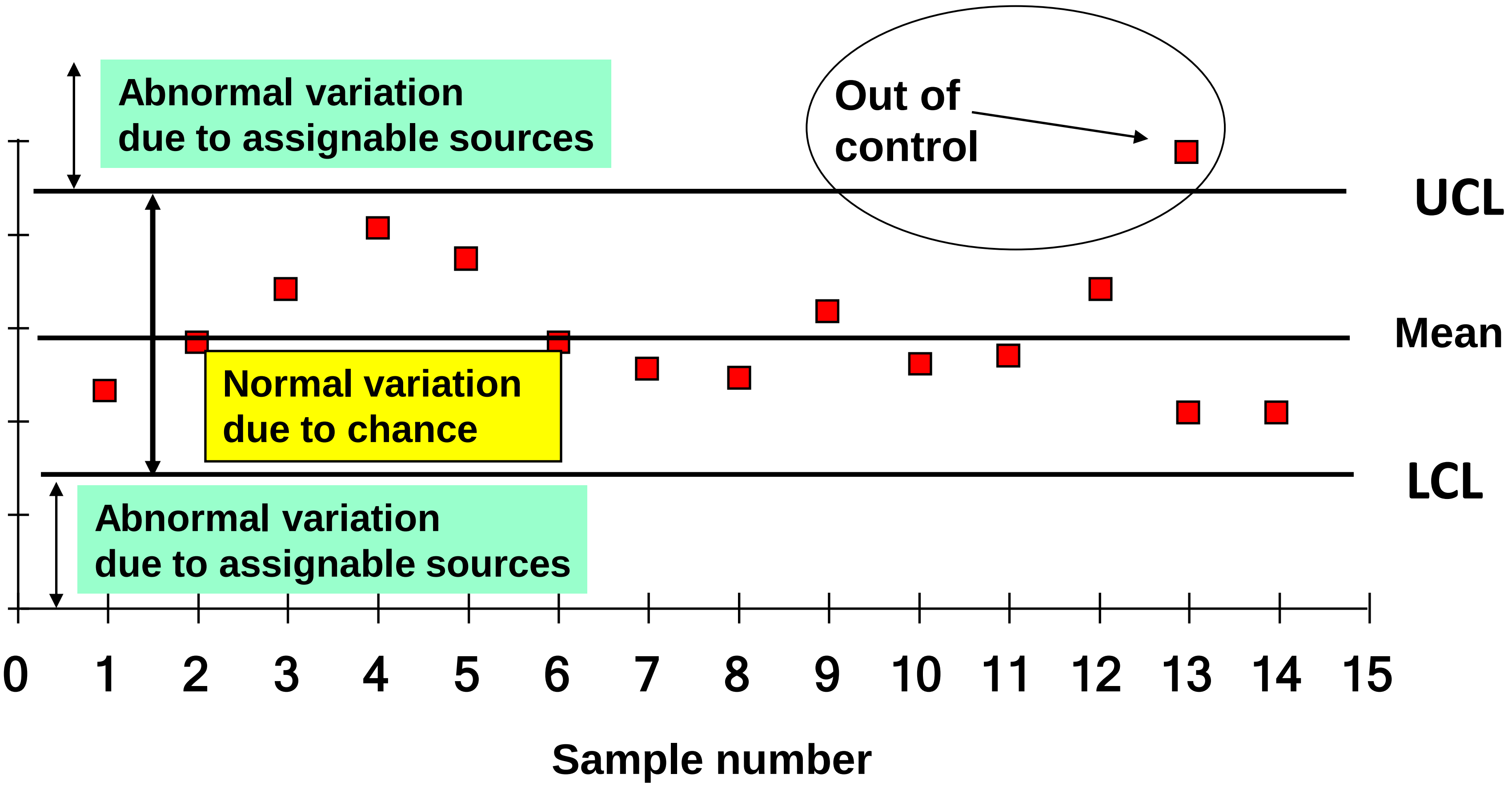
- Process control: concerned with monitoring quality while the product or service is being produced
- Statistical process control: testing a sample of output to determine if the process is producing items within a preselected range
- Control charts: run charts to which two horizontal lines (called control limits) are added to monitor process output to see if it is random
- Control limits: chosen statistically to provide a high probability that points will fall between these limits if the process is in control
  - Lower Control Limit (LCL)
  - Upper Control Limit (UCL)
- System stability
  - Stable system: a system governed only by common causes
  - In control: if no special causes affect the output of the process
  - Out of control: when special causes are present in the process

Control Chart



Variation

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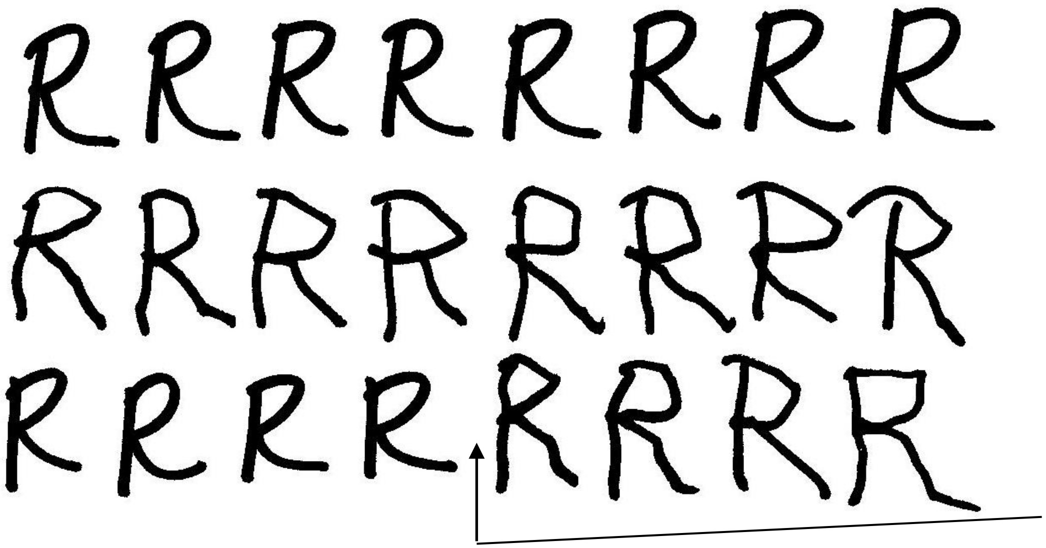
- Processes usually exhibit some variation in their output
  - Common (or natural) variation: variation that is inherent in the process itself
  - Assignable variation: variation that is caused by factors that can be identified and managed
- It is generally accepted that as variation is reduced, quality is improved



Common-cause variation ("normal" hand)

Common-cause variation (the "other" hand)

Assignable variation (change from "normal" to "other" hand)

Constructing Control Charts

1. Preparation
   - Choose the metric to be monitored
   - Determine the basis, size, and frequency of sampling
   - Set up the control chart
2. Data collection
   - Record the data
   - Calculate relevant statistics: averages, ranges, proportions, and so on
   - Plot the statistics on the chart
3. Determination of trial control limits
   - Draw the center line (process average) on the chart
   - Compute the upper and lower control limits
4. Analysis and interpretation
   - Investigate the chart for lack of control
   - Eliminate out-of-control points
   - Recompute control limits if necessary
5. Use as a problem-solving tool
   - Continue data collection and plotting
   - Identify out-of-control situations and take corrective action
6. Determination of process capability using the control chart data

Quality Metrics

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- Continuous metric: one that is calculated from data that is MEASURED as the degree of conformance to a specification on a continuous scale of measurement
  - Examples: Length, volume, weight, height, time, color (spectrum), sound pitch (frequency), sound intensity or noise level (dB/square meter)
- Discrete metric: one that is calculated from data that is COUNTED
  - Examples: Number of flaws, years, accidents, students, satisfied/dissatisfied, good/bad

Control Chart Classification

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Type of Control Chart:

- **Control Chart**
  - **Continuous metric** (measured)
    - Mean chart ($\overline{x}$ (“x-bar”) chart) [detect the shift of the mean]
    - Range chart (R-chart) [detect the shift of dispersion]
  - **Discrete metric** (counted)
    - P-chart [Fraction defectives]
    - C-chart [# of defects]

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    - C-chart [# of defects]

Continuous Metrics

- Continuous Metrics are *measured*
  - X-bar charts: a plot of the means of the samples taken from a process
    - Used to monitor the central tendency (*mean*) of a process
  - R-chart: a plot of the *range* within each sample
    - The range is the difference between the highest and lowest numbers in each sample
    - Used to monitor the process dispersion

X-Bar and R Chart Equations

- X-Bar = $\bar{X}$ chart

Control limits: $CL = \bar{\bar{X}} \pm A_2 \bar{R}$

Upper control limit: $UCL = \bar{\bar{X}} + A_2 \bar{R}$

Lower control limit: $LCL = \bar{\bar{X}} - A_2 \bar{R}$

- $\bar{\bar{X}}$ = average of sample means
  = “Grand Mean” (“X-dbl-bar”)
- $\bar{R}$ = average range for all samples
- A_2 = constant from Table S6.1

- R-Chart

Upper control limit: $UCL = D_4 \bar{R}$

Lower control limit: $LCL = D_3 \bar{R}$

- D_3 and D_4 = constants from Table S6.1

Example for X-Bar Chart and R-Chart

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- A machine operator has obtained the following measurements:

Sample — k # samples = 4

n sample size = 5

1	2	3	4
4.5	4.6	4.5	4.7
4.2	4.5	4.6	4.6
4.3	4.4	4.4	4.8
4.3	4.7	4.4	4.5
4.3	4.3	4.6	4.9

- Create R-Chart and X-Bar Charts and determine if process is in or out of control

R-Chart Solution

Sample			
1	2	3	4
4.5	4.6	4.5	4.7
4.2	4.5	4.6	4.6
4.3	4.4	4.4	4.8
4.3	4.7	4.4	4.5
4.3	4.3	4.6	4.9

- Calculate range of each sample (range = largest – smallest)
  - R1 = 4.5 – 4.2 = 0.3
  - R2 = 4.7 - 4.3 = 0.4
  - R3 = 4.6 – 4.4 = 0.2
  - R4 = 4.9 – 4.5 = 0.4
- So, R-bar = average range for all samples
  - R-bar = (R1 + R2 + R3 + R4) / 4
  - R-bar = (0.3 + 0.4 + 0.2 + 0.4) / 4 = 0.325
- Calculate upper and lower control limits
  - UCL = (D4)R-bar
  - LCL = (D3)R-bar
  - Look up D3 and D4 on Table S6.1

Control Chart Factors (Table S6.1)

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n = sample size
k = number of samples

Sample Size n	Mean Factor A_2	Upper Range D_4	Lower Range D_3
2	1.880	3.268	0
3	1.023	2.574	0
4	0.729	2.282	0
5	0.577	2.115	0
6	0.483	2.004	0
7	0.419	1.924	0.076
8	0.373	1.864	0.136
9	0.337	1.816	0.184
10	0.308	1.777	0.223
12	0.266	1.716	0.284

R-Chart Solution

Sample			
1	2	3	4
4.5	4.6	4.5	4.7
4.2	4.5	4.6	4.6
4.3	4.4	4.4	4.8
4.3	4.7	4.4	4.5
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- Calculate range of each sample (range = largest – smallest)
  - R1 = 4.5 – 4.2 = 0.3
  - R2 = 4.7 - 4.3 = 0.4
  - R3 = 4.6 – 4.4 = 0.2
  - R4 = 4.9 – 4.5 = 0.4
- So, R-bar = average range for all samples
  - R-bar = (R1 + R2 + R3 + R4) / 4
  - R-bar = (0.3 + 0.4 + 0.2 + 0.4) / 4 = 0.325
- Calculate upper and lower control limits
  - UCL = (D4)R-bar = (2.115)(0.325) = 0.687
  - LCL = (D3)R-bar = 0(0.325) = 0
- Plot and evaluate

R-Chart Solution

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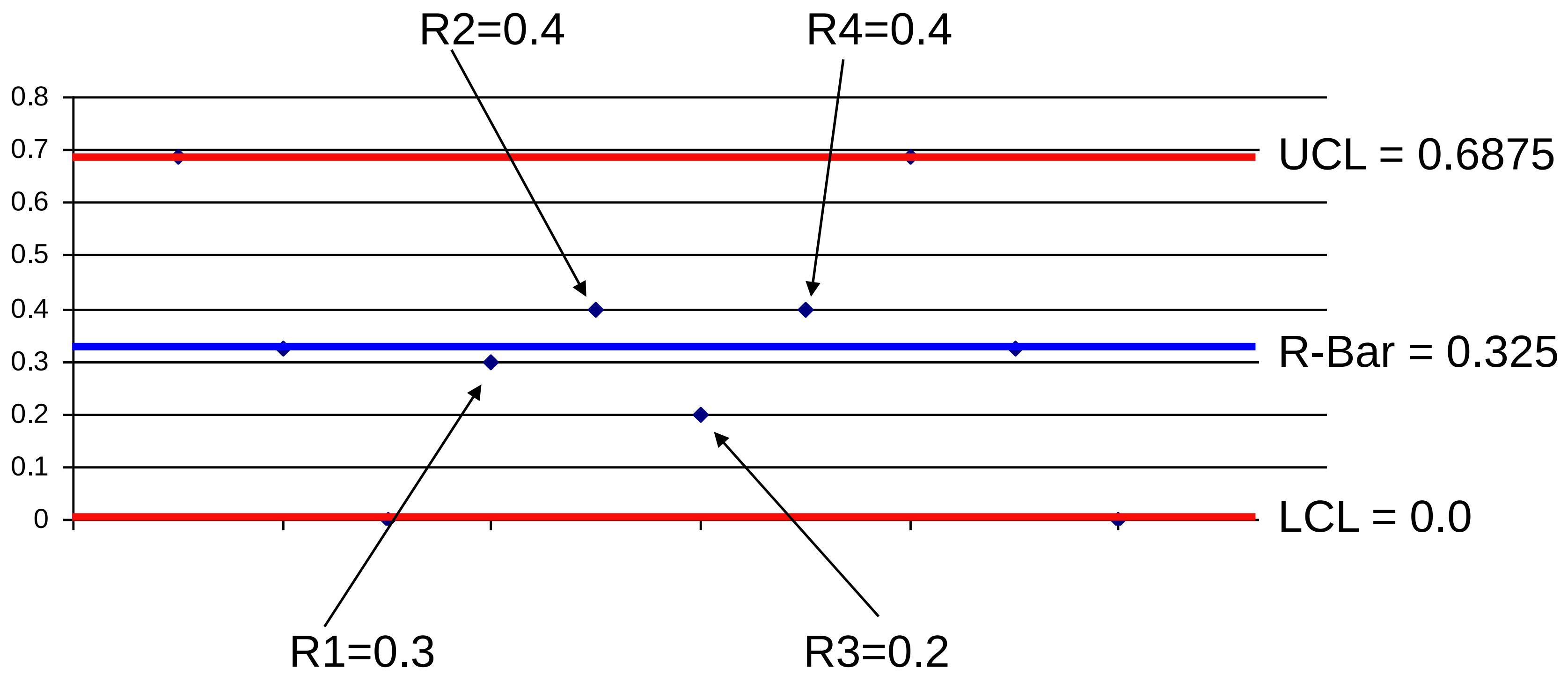

X-Bar Chart Solution

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Sample			
1	2	3	4
4.5	4.6	4.5	4.7
4.2	4.5	4.6	4.6
4.3	4.4	4.4	4.8
4.3	4.7	4.4	4.5
4.3	4.3	4.6	4.9

- Calculate X-bar for each sample
  - X-bar1 = (4.5 + 4.2 + 4.3 + 4.3 + 4.3) / 5 = 4.32
  - X-bar2 = (4.6 + 4.5 + 4.4 + 4.7 + 4.3) / 5 = 4.50
  - X-bar3 = 4.50
  - X-bar4 = 4.70
- X-dbl-bar = Grand Mean = avg of sample means
  - X-dbl-bar = (x-bar1 + x-bar2 + x-bar3 + x-bar4) / 4
  - X-dbl-bar = (4.32 + 4.50 + 4.50 + 4.70) / 4 = 4.50
- Calculate control limits
  - UCL = X-dbl-bar + A2(R-bar)
  - LCL = X-dbl-bar - A2(R-bar)
  - Look up A2 on Table S6.1

Control Chart Factors (Table S6.1)

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X-Bar Chart Solution

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Sample			
1	2	3	4
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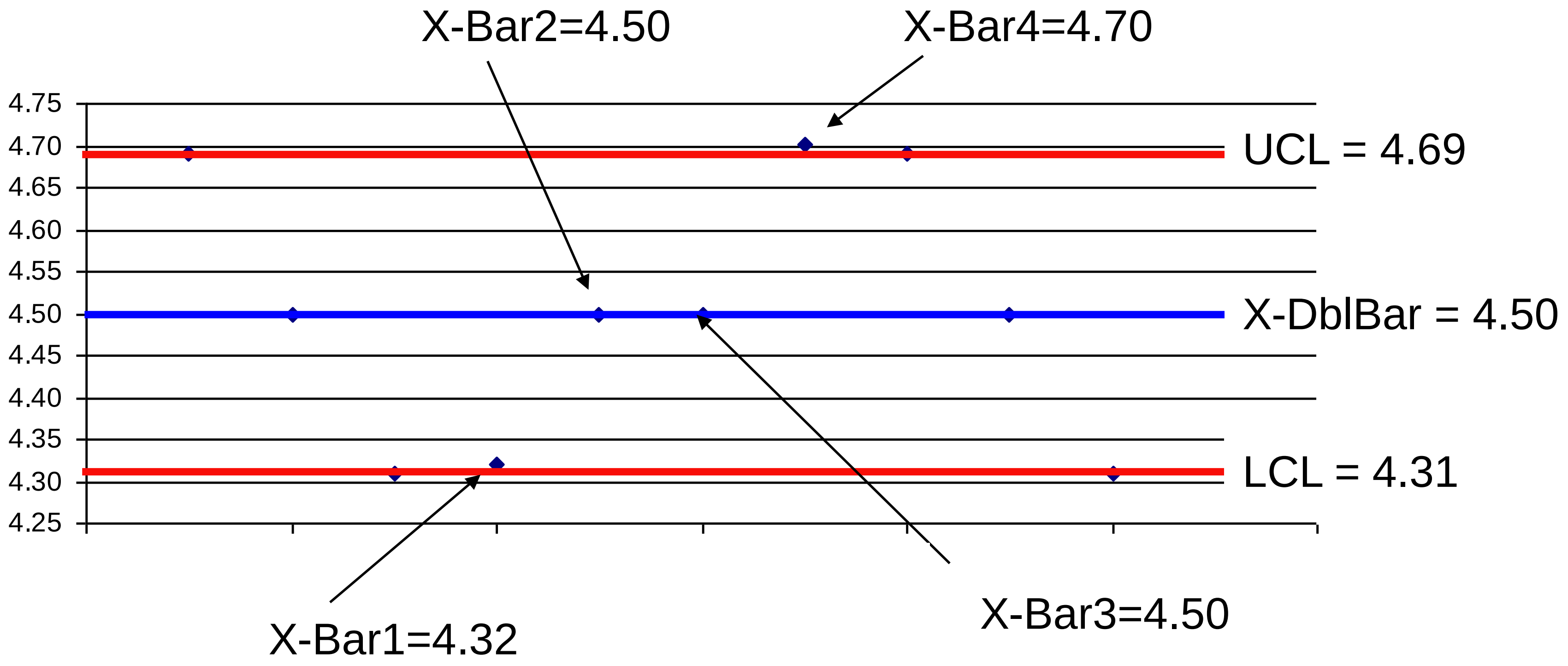
- Calculate X-bar for each sample
  - X-bar1 = (4.5 + 4.2 + 4.3 + 4.3 + 4.3) / 5 = 4.32
  - X-bar2 = (4.6 + 4.5 + 4.4 + 4.7 + 4.3) / 5 = 4.50
  - X-bar3 = 4.50
  - X-bar4 = 4.70
- X-dbl-bar = Grand Mean = avg of sample means
  - X-dbl-bar = (x-bar1 + x-bar2 + x-bar3 + x-bar4) / 4
  - X-dbl-bar = (4.32 + 4.50 + 4.50 + 4.70) / 4 = 4.50
- Calculate control limits
  - UCL = X-dbl-bar + A2(R-bar) = 4.50 + (0.577)(0.325) = 4.69
  - LCL = X-dbl-bar - A2(R-bar) = 4.50 - (0.577)(0.325) = 4.31
- Plot and evaluate

X-Bar Solution



Control Chart Classification

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  - **Discrete metric** (counted)
    - P-chart [Fraction defectives]
    - C-chart [# of defects]

Discrete Metrics

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- Discrete Metrics are *counted*
  - p-Chart: Control chart used to monitor the percent (or *proportion*) of defectives in a process
    - When observations can be placed into two categories
      - Examples: good or bad, pass or fail, operate or don't operate
    - When the data consists of multiple samples of several observations each
  - c-Chart: Control chart used to monitor the *number* of defects per unit
    - Used when not possible to compute population percentages
    - Use only when the number of occurrences per unit of measure can be counted; non-occurrences cannot be counted
      - Examples: scratches, chips, dents, or errors per item; cracks or faults per unit of distance; breaks or tears per unit of area; bacteria or pollutants per unit of volume; calls, complaints, failures per unit of time

p-Chart Equations

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z = # standard deviations for 99.7% confidence
We will use z = 3

Control limits: $CL = \bar{p} \pm z\sigma_p = \bar{p} \pm 3\sigma_p$

Upper control limit: $UCL = \bar{p} + z\sigma_p = \bar{p} + 3\sigma_p$

Lower control limit: $LCL = \bar{p} - z\sigma_p = \bar{p} - 3\sigma_p$

$\bar{p}$ = average % defective in sample = $\dfrac{\text{total \# defectives}}{\text{total \# observations}}$

total # observations = (# samples)(sample size) = kn
k = number of samples, n = sample size

$$\sigma_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$ = estimate of standard deviation

Note: if LCL is negative, zero should be used as a lower limit
If varying sample sizes, use the sample average

Creating p-Charts

1. Calculate the sample proportions p for each sample
2. Calculate the average of the sample proportions
3. Calculate the standard deviation of the sample proportion
4. Calculate the control limits
5. Plot the individual sample proportions, the average of the proportions, and the control limits

p-Chart Example

- Using a popular database software package, data entry clerks at ARCO key in thousands of insurance claims each day. One hundred entered records are carefully examined to make sure they contained no errors. The fraction defective in each sample was then computed. Samples of the work of the recent 20 days is shown below. Establish a proper control chart to make sure the data entry process is in control.

Day	# incorrect records	Day	# incorrect records
1	6	11	6
2	5	12	1
3	0	13	8
4	1	14	7
5	4	15	5
6	2	16	4
7	5	17	11
8	3	18	3
9	3	19	0
10	2	20	4

p-Chart Example

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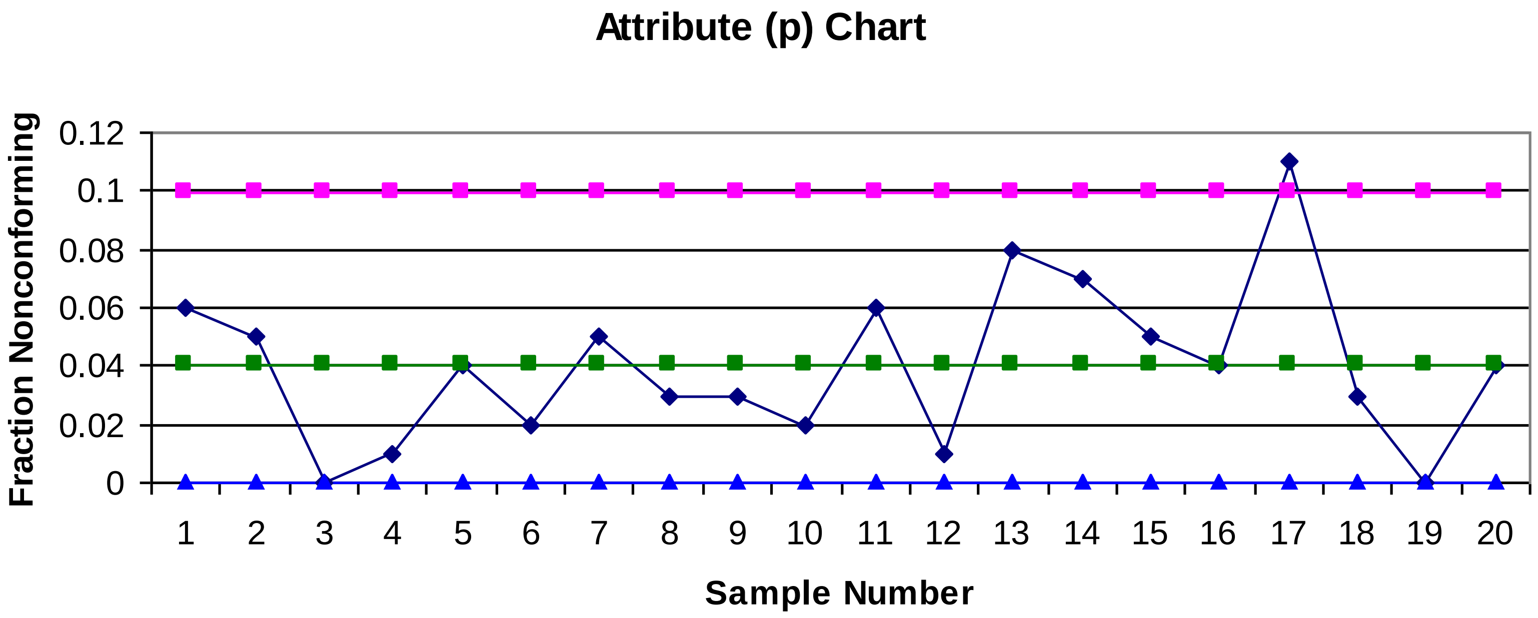
- P-bar = avg % defectives = tot # defectives/tot # observations
- P-bar = tot # defectives/kn
- k=# samples=20, n=sample size=100
- P-bar = 80 defectives/(20 samples)(100 records/sample) = 0.040
- σ_p = est of std deviation

$$\sigma_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \sqrt{\frac{0.040(1-0.040)}{100}} = 0.020$$

- Calc control limits
  - UCL = p-bar + $3\sigma_p$ = 0.040 + 3(0.020) = 0.099 = 0.10
  - LCL = p-bar - $3\sigma_p$ = 0.040 - 3(0.020) = -0.019 → 0
- Plot UCL, LCL and fraction defectives
- Fraction defective = # non-conform/sample / n(sample size)
  - P1 = 6/100 = 0.06, P2 = 5/100 = 0.05, etc.

p-Chart Example

Sample Number	# of Errors	Fraction Defective	Sample Number	# of Errors	Fraction Defective
1	6	0.06	11	6	0.06
2	5	0.05	12	1	0.01
3	0	0.00	13	8	0.08
4	1	0.01	14	7	0.07
5	4	0.04	15	5	0.05
6	2	0.02	16	4	0.04
7	5	0.05	17	11	0.11
8	3	0.03	18	3	0.03
9	3	0.03	19	0	0.00
10	2	0.02	20	4	0.04



c-Chart Equations

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- Where p-chart monitors the proportion of nonconforming items, a c-chart monitors the “number of nonconformances” per unit (i.e., a count of the number of defects, errors, failures, etc.)
- These charts are used extensively in service organizations

$\bar{c}$ = average number of defects = total defects/k

Control limits: $$CL = \bar{c} \pm z\sqrt{\bar{c}} = \bar{c} \pm 3\sqrt{\bar{c}}$$

Upper control limit: $$UCL = \bar{c} + z\sqrt{\bar{c}} = \bar{c} + 3\sqrt{\bar{c}}$$

Lower control limit: $$LCL = \bar{c} - z\sqrt{\bar{c}} = \bar{c} - 3\sqrt{\bar{c}}$$

z = # standard deviations for 99.73% confidence
We will use z = 3

Note: if LCL is negative, zero should be used as a lower limit

c-Chart Example

- Red Top Cab Company receives several complaints per day about the behavior of its drivers. Over a twenty-five day period (where days are the unit of measure) the owner received the number of calls from irate passengers. Calculate control limits and determine if this process is in or out of control.

Day	Complaints	Day	Complaints	Day	Complaints
1	2	10	2	19	0
2	1	11	0	20	2
3	1	12	0	21	8
4	0	13	4	22	7
5	5	14	3	23	0
6	2	15	1	24	1
7	3	16	3	25	2
8	1	17	1		
9	1	18	1		

c-Chart Example

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- Complaints = defects
- C-bar = avg # defects = total defects/# samples
  - C-bar = 51 complaints/25 days = 2.04
- Control limits

$$\mathrm{UCL} = \bar{c} + z\sqrt{\bar{c}} = \bar{c} + 3\sqrt{\bar{c}} = 2.04 + 3\sqrt{2.04} = 2.04 + 4.285 = 6.325$$

$$\mathrm{LCL} = \bar{c} - z\sqrt{\bar{c}} = \bar{c} - 3\sqrt{\bar{c}} = 2.04 - 3\sqrt{2.04} = 2.04 - 4.285 = -2.245 = 0$$

- Plot UCL, LCL, c-bar and defects/day

c-Chart Example

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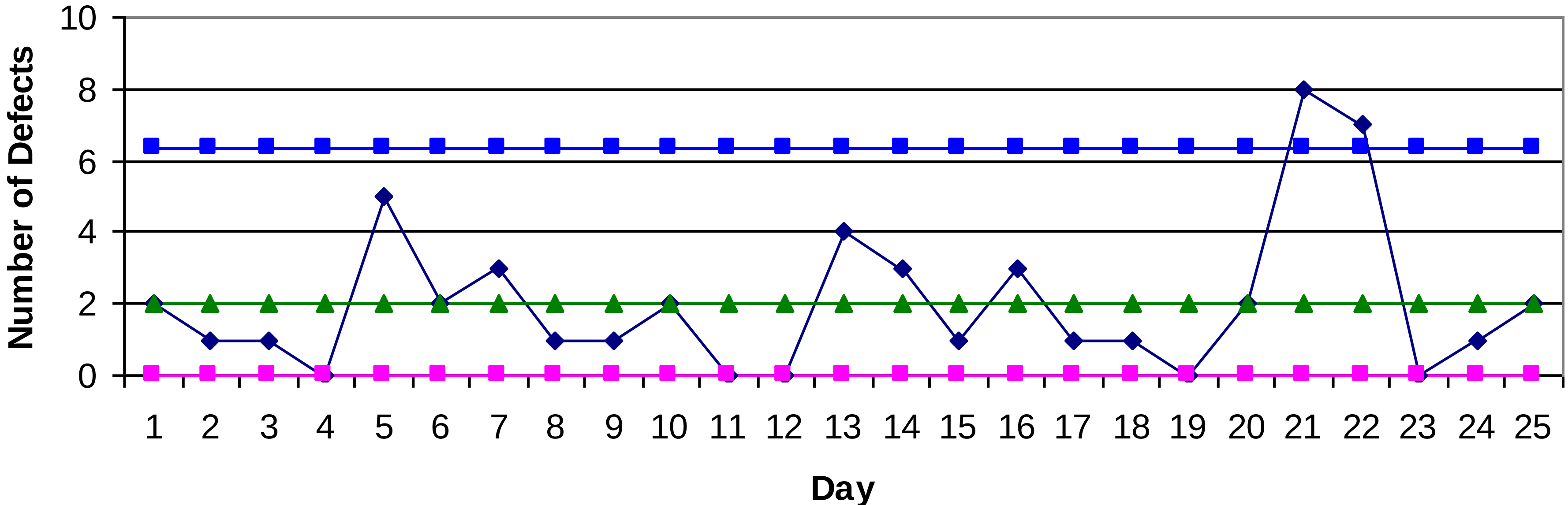
Day	Complaints	Day	Complaints	Day	Complaints
1	2	10	2	19	0
2	1	11	0	20	2
3	1	12	0	21	8
4	0	13	4	22	7
5	5	14	3	23	0
6	2	15	1	24	1
7	3	16	3	25	2
8	1	17	1		
9	1	18	1		

Attribute (c) Chart



Process Capability

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- Specification (or tolerance) limits: Range of acceptable values established by engineering design or customer requirements
  - LSL = Lower Specification Limit
  - USL = Upper Specification Limit

- Process Capability (C_p): the determination of whether the variability inherent in the output of a process that is in control falls within the acceptable range of variability allowed by the design specifications
  - If it is within specifications, the process is "capable"

- For a process to be deemed capable, it must have a capability ratio of at least 1.0 (however 1.0 would mean the process is just barely capable)
  - Current trend is for $C_p \geq 1.33$ (our criteria in this class!)
  - (Note: for "six sigma": $C_p = 2.0$)

Process Capability

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- Process capability is the natural variation in a process that results from common causes:
  A. $C_p \geq 1.33$: indicates good capability as in (A, next chart)
     - Many firms require C_p values of 1.66 or greater from their suppliers, which equates to a tolerance range of about 10 standard deviations
     - Our criteria: $C_p \geq 1.33$ = capable, $C_p < 1.33$ = not capable

  B. $C_p = 1$: the natural variation is the same as the design specification width, as in (B)

  C. $C_p < 1$: a significant percentage of output will not conform to the specifications as in (C)

  D. The value of C_p does not depend on the mean of the process; thus, a process may be off-center, such as in (d), and still show an acceptable value of C_p (D)

Process Capability versus Design Specifications

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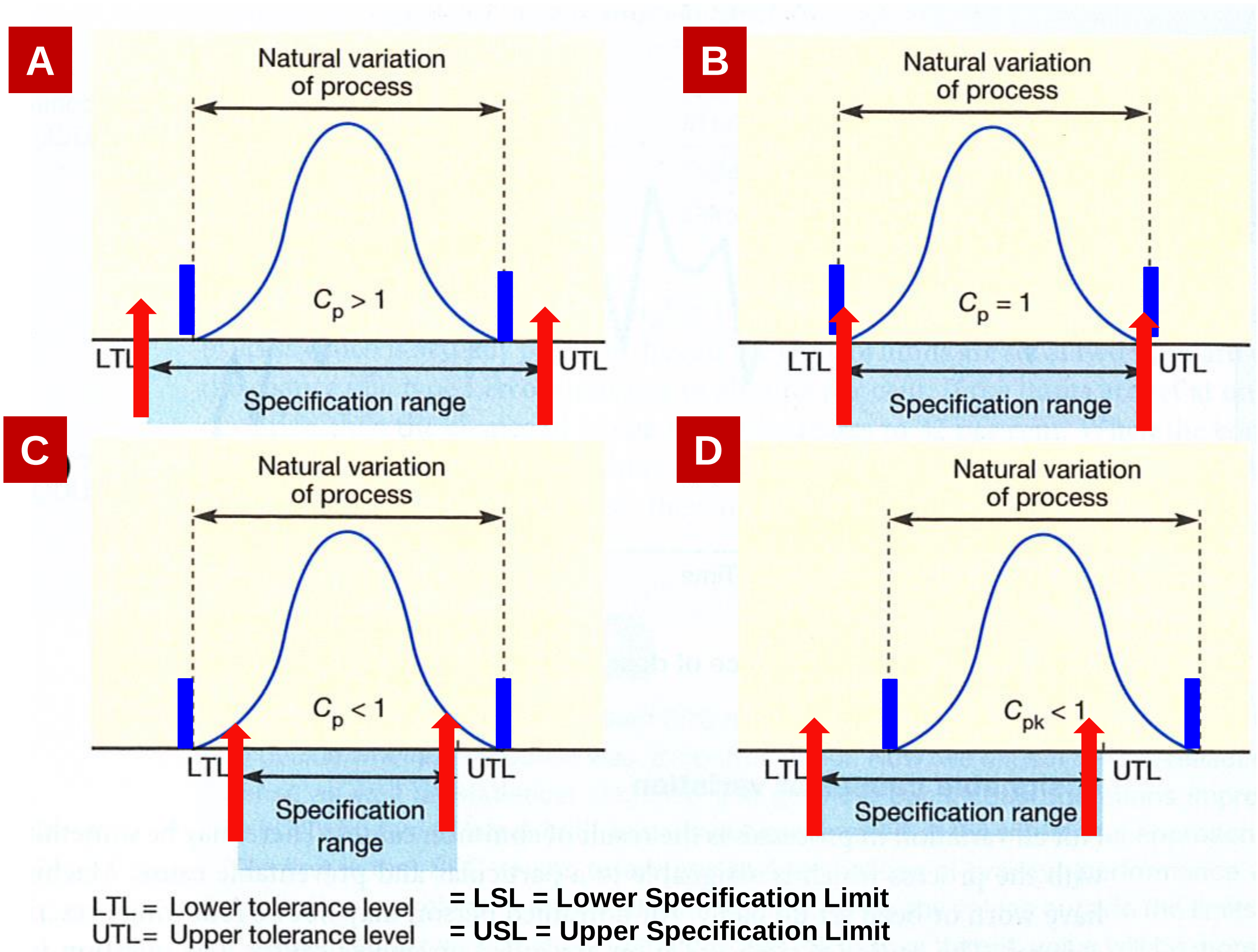

= LSL = Lower Specification Limit
= USL = Upper Specification Limit

Process Capability, C_p

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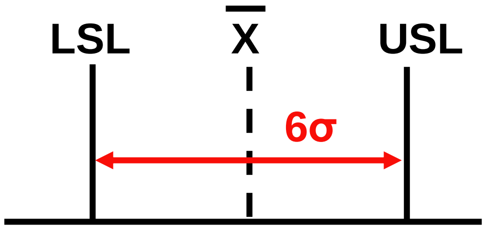
- Process capability (C_p):

$$C_p = \frac{(USL - LSL)}{6\sigma}$$



Where:

USL = Upper Specification Limit

LSL = Lower Specification Limit

σ = standard deviation of the process

C_p $\geq$ 1.33 is capable (our criteria in this class)

= 1.00 process is barely capable (but not good enough)

< 1.00 process is not capable

Process Capability Example

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$\sigma = 3.05$

USL = 32.0

LSL = 28.0

What is the process capability?

$$Cp = \frac{(USL - LSL)}{6(sigma)}$$

$$Cp = \frac{(32 - 28)}{6(3.05)} = 0.2185$$

Off-Centered Processes, C_{pk}

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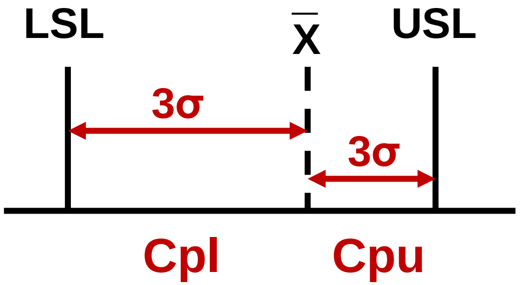
- Shows how well the parts being produced fit into the range specified by the design specifications

$$C_{pk} = \min\left(\underbrace{\frac{\overline{X} - LSL}{3\sigma}}_{Cpl} \text{ or } \underbrace{\frac{USL - \overline{X}}{3\sigma}}_{Cpu}\right)$$



- where

USL = Upper Specification Limit

LSL = Lower Specification Limit

$\overline{X}$ = the mean performance of the process

σ = standard deviation of the process

Cpl = Cp lower
Cpu = Cp upper

C_{pk} ≥ 1.33 is capable (our criteria in this class)
= 1.00 process is barely capable (but not good enough)
< 1.00 process is not capable

Solved Problem

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- Suppose that a specification calls for LSL = 2.0 and USL = 6.0. A sample of 100 parts found x-bar = 4.5 and σ = 0.5. Calculate Cp and Cpk and evaluate if the process is capable or not?

Calculate Cp:
X-bar = mean = 4.5
Sigma = 0.5
LSL = 2.0, USL = 6.0

$$Cp: \frac{USL - LSL}{6(sigma)}$$

$$Cp = \frac{6.0 - 2.0}{6(0.5)} = \mathbf{1.33} = \text{CAPABLE?}$$

Calculate Cpk:

$$Cpl: \frac{\text{X-bar} - LSL}{3(sigma)} = \frac{4.5 - 2.0}{3(0.5)} = \mathbf{1.667} = \text{CAPABLE?}$$

$$Cpu: \frac{USL - \text{X-bar}}{3(sigma)} = \frac{6.0 - 4.5}{3(0.5)} = \mathbf{1.00} = \text{NOT CAPABLE!}$$

Therefore, Cpk = **min** (Cpl and Cpu)….**Cpk = 1.00** = NOT CAPABLE