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Chapter 4: Forecasting

Learning Objectives

- Understand the three-time horizons and which models apply for each use
- Explain when to use each of the four qualitative models
- Understand the various quantitative models, including the naive, moving average, exponential smoothing, and trend methods
- Compute three measures of forecast accuracy
- Develop seasonal indexes
- Conduct a regression and correlation analysis

Forecasting and Demand Planning

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- Forecasting: the art and science of predicting future events
  - Many firms integrate forecasting with supply chain and capacity management systems to make better operational decisions
  - Accurate forecasts are needed throughout the supply chain, and are used by all functional areas of the organization, including accounting, finance, marketing, operations, and distribution

- Demand planning software: systems that integrate marketing, inventory, sales, operations planning, and financial data to synchronize the supply chain

Forecasting and Demand Planning

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- Planning horizon: the length of time on which a forecast is based

- Short range forecasts = Up to 1 year, but generally less than 3 months
  - Ex: Purchasing, job scheduling, workforce levels, inventory

- Medium range forecasts = 3 months to 3 years
  - Ex: Sales & production planning, budgeting

- Long range forecasts = 3+ years
  - Ex: Facility locations, R&D, new product launches

Types of Forecasts

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- Economic forecasts: planning indications that are valuable in helping organizations prepare medium- to long-range forecasts
  - Address business cycle – inflation rate, money supply, housing starts, etc.
- Technological forecasts: long-term forecasts concerned with the rates of technological progress
  - Impacts development of new products
- Demand forecasts: projections of a company's sales for each time period in the planning horizon
  - Forecasts of demand drive decisions in many areas, examples:
    - Supply-Chain Management – good supplier relations, advantages in product innovation, cost and speed to market
    - Human Resources – hiring, training, laying off workers
    - Capacity – capacity shortages can result in undependable delivery, loss of customers, loss of market share

Features of Forecasts

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- Assumes historical system: past → future
- Forecasts are rarely perfect because of randomness
- Forecasts more accurate for groups vs. individuals
- Forecast accuracy decreases as time horizon increases

Eight* Steps in Forecasting

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1. Determine the use of the forecast
2. Select the items to be forecast
3. Determine the time horizon of the forecast
4. Select the forecasting model(s)
5. Gather the data
6. Make the forecast
7. Validate and implement results

* *The text only gives the first seven steps, but we also need to:*

8. Monitor the forecasting performance

Basic Forecasting Methods

Forecasting Methods

- **Qualitative Methods**
  - Jury of executive opinion
  - Delphi method
  - Sales force composite
  - Market survey
- **Quantitative Methods**
  - **Time Series**
    - Naïve
    - Moving averages
    - Weighted averages
    - Exponential smoothing
    - Trend projection
  - **Regression Methods**
    - Linear regression
    - Multiple regression

Qualitative (Judgmental) Forecasting

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- Qualitative forecasts: forecasts that incorporate such factors as the decision maker's intuition, emotions, personal experiences and value system
  - Jury of executive opinion: a forecasting technique that uses the opinion of high-level managers to form a group estimate of demand
  - Delphi method: a forecasting technique using a group process that allows experts to make forecasts
  - Sales force composite: a forecasting technique based on salespersons' estimates of expected sales
  - Consumer Market Survey: a forecasting method that solicits input from customers or potential customers regarding future purchasing plans

Examples:
Jury of Exec opinion = the Fed
Delphi method = Take a poll, narrow it down
Sales force composite = ask Bob in Sales
Consumer Survey = cold call or email you

Qualitative (Judgmental) Forecasting

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- When no historical data is available, only judgmental forecasting is possible
- The major reasons for using judgmental methods are:
  - Greater accuracy (*if there is no historical data)
  - Ability to incorporate unusual or one-time events
  - The difficultly of obtaining the data necessary for quantitative techniques

Basic Forecasting Methods

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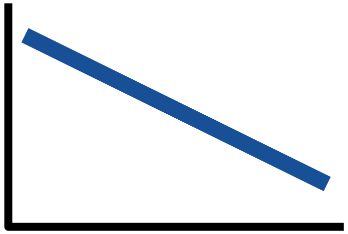
Forecasting Methods

- **Qualitative Methods**
  - Jury of executive opinion
  - Delphi method
  - Sales force composite
  - Market survey
- **Quantitative Methods**
  - **Time Series**
    - Naïve
    - Moving averages
    - Weighted averages
    - Exponential smoothing
    - Trend projection
  - **Regression Methods**
    - Linear regression
    - Multiple regression

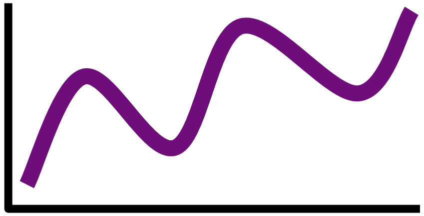
Decomposition of a Time Series

- Time series: a forecasting technique that uses a series of *past* data points to make a forecast
  - Analyze by breaking down into components and projecting them forward – there are four components:

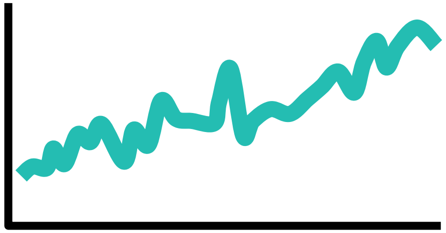
1. Trend: the underlying pattern of growth or decline
2. Cyclical patterns: regular patterns in a data series that take place over long periods of time
3. Random variation (or noise): unexplained deviation from a predictable pattern
4. Seasonal patterns: repeatable periods of ups and downs over short periods of time



Trend



Cycle



Random Variation

Seasonal Patterns

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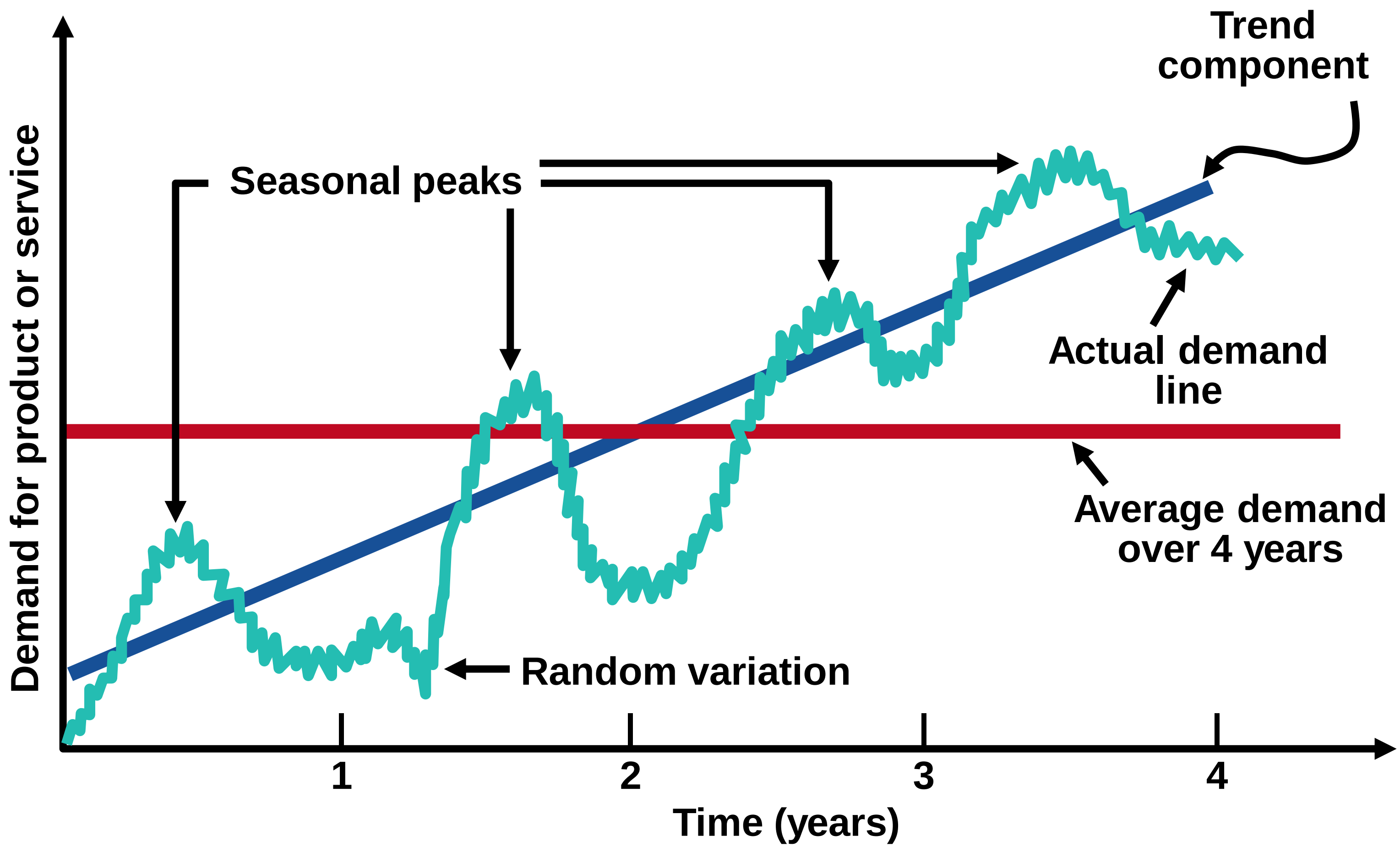
- There are six common seasonality patterns

Period Length	“Season” Length	Number of “Seasons” in Pattern
Week	Day	7
Month	Week	4
Month	Day	30
Year	Quarter	4
Year	Month	12
Year	Week	52

Demand with Growth Trend and Seasonality Over Four Years



Forecasting in Practice

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- Managers use a variety of judgmental and quantitative forecasting techniques
- Statistical methods alone cannot account for such factors as sales promotions, competitive strategies, unusual economic disturbances, new products, large one-time orders, natural disasters, or labor complications
- The first step in developing a practical forecast is to understand the purpose, time horizon, and level of aggregation
- Different forecasting methods require different levels of technical ability and understanding of mathematical principles and assumptions

Naïve Forecasts

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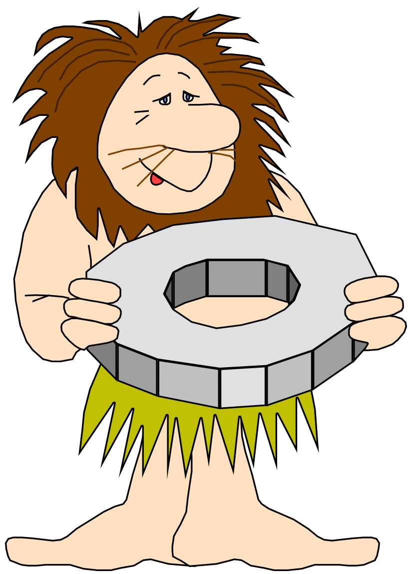
- Naïve forecast: The forecast for any period equals the previous period's actual value

- Benefits
  - Simple to use
  - Virtually no cost
  - Quick and easy to prepare
  - Data analysis is nonexistent
  - Easily understandable
  - Can provide high accuracy
  - Can be good starting point



Uh, give me a minute…
We sold 250 wheels last week...
So next week we should sell
________ wheels?

Moving Average Forecasts

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- Moving Average (MA) – A technique that averages a number of recent actual values, updated as new values become available
- MA methods work best for short planning horizons when there is no major trend, seasonal, or business cycle pattern
- As the value of “n” increases, the forecast reacts slowly to recent changes in the time series data

$$\text{Moving average} = \text{MA}(n) = \frac{\Sigma \text{ Demand in previous } n \text{ periods}}{n}$$

Where:

n = number of periods (data points) in the moving average

Moving Average Example

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- Donna's Garden Supply wants a 3-month, MA(3) and a 6-month, MA(6) moving average forecast for storage sheds, including a forecast for next January

- Storage shed sales are shown below

Month	Actual Sales
January	10
February	12
March	13
April	16
May	19
June	23
July	26
August	30
September	28
October	18
November	16
December	14
January	

Moving Average Example

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MONTH	ACTUAL SHED SALES	3-MONTH MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	$(10 + 12 + 13)/3 = 11\ ^2/_3$
May	19	$(12 + 13 + 16)/3 = 13\ ^2/_3$
June	23	$(13 + 16 + 19)/3 = 16$
July	26	$(16 + 19 + 23)/3 = 19\ ^1/_3$
August	30	$(19 + 23 + 26)/3 = 22\ ^2/_3$
September	28	$(23 + 26 + 30)/3 = 26\ ^1/_3$
October	18	$(26 + 30 + 28)/3 = 28$
November	16	$(30 + 28 + 18)/3 = 25\ ^1/_3$
December	14	$(28 + 18 + 16)/3 = 20\ ^2/_3$

Donna's Garden Supply: MA(3)

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$$\text{Moving average} = MA(n) = \frac{\Sigma \text{ demand in previous } n \text{ periods}}{n}$$

Month	Actual Sales	MA(3) Forecast
January	10	
February	12	
March	13	
April	16	11.67
May	19	13.67
June	23	16.00
July	26	19.33
August	30	22.67
September	28	26.33
October	18	28.00
November	16	25.33
December	14	20.67
January		16.00

A = Actual Sales (Demand)
F = Forecast Sales

F(April) = [A(Mar)+A(Feb)+A(Jan)]/3

F(April) = [13+12+10]/3 = 11.67

F(May) = [A(Apr)+A(Mar)+A(Feb)]/3

F(May) = [16+13+12]/3=13.67

F(Jan) = [A(Dec)+A(Nov)+A(Oct)]/3

F(Jan) = [14+16+18]/3 = 16.00

Donna's Garden Supply: MA(6)

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Month	Actual Sales	MA(6) Forecast
January	10	
February	12	
March	13	
April	16	
May	19	
June	23	
July	26	15.50
August	30	18.17
September	28	21.17
October	18	23.67
November	16	24.00
December	14	23.50
January		22.00

A = Actual Sales (Demand)
F = Forecast Sales

$$\text{Moving average} = \text{MA(n)} = \frac{\Sigma \text{ demand in previous n periods}}{n}$$

F(July) = [A(Jun)+A(May)+A(Apr)+A(Mar)+A(Feb)+A(Jan)]/6

F(July) = [23+19+16+13+12+10]/6 = 15.5

F(Jan) = [A(Dec)+A(Nov)+A(Oct)+A(Sep)+A(Aug)+A(Jul)]/6

F(Jan) = [14+16+18+28+30+26]/6 = 22.00

Weighted Moving Average Forecasts

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- Weighted Moving Average (WMA): *more recent values* in a series are given *more weight* in computing the forecast
- The moving average formula implies an *equal weight* being placed on each value that is being averaged
- Choosing weights
  - Experience and trial-and-error are the simplest ways
  - Generally, the most recent past is the best indicator of what to expect in the future, and therefore, should get higher weighting
  - Typically, the weights are given

$$\text{Weighted Moving Average} = \frac{\Sigma\,(\text{Weight for period } n)(\text{Demand in period } n)}{\Sigma\ \text{Weights}}$$

Weighted Moving Average Example

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- Donna's Garden Supply wants to forecast storage shed sales by weighting the past 3 months with more weight given to recent data to make them more significant

$$\text{Weighted Moving Average} = \frac{\Sigma\,(\text{Weight for period } n)(\text{Demand in period } n)}{\Sigma\,\text{Weights}}$$

Month	Actual Sales
January	10
February	12
March	13
April	16
May	19
June	23
July	26
August	30
September	28
October	18
November	16
December	14
January	

Weights Applied	Period
3	Last month
2	Two months ago
1	Three months ago
6	Sum of weights

Donna's Garden Supply: WMA

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$$\text{Weighted moving average} = \frac{\Sigma\,(\text{Weight for period } n)(\text{Demand in period } n)}{\Sigma\,\text{Weights}}$$

Month	Actual Sales	WMA Forecast
January	10	
February	12	
March	13	
April	16	12.17
May	19	14.33
June	23	17.00
July	26	20.50
August	30	23.83
September	28	27.50
October	18	28.33
November	16	23.33
December	14	18.67
January		15.33

A = Actual Sales (Demand)
F = Forecast Sales

Weights Applied	Period
3	Last month
2	Two months ago
1	Three months ago
6	Sum of weights

F(April) = [3*A(Mar)+2*A(Feb)+1(A-Jan)]/6

F(April) = [(3*13) +(2*12) +(1*10)]/6 = 12.17

F(Jan) = [3*A(Dec)+2*A(Nov)+1*A(Oct)]/6

F(Jan) = [(3*14) +(2*16) +(1*18)]/6=15.33

Potential Problems with Moving Averages

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- Increasing “n” smooths the forecast but makes it less sensitive to changes
- Does not forecast trends well, they lag the actual values
- Requires extensive records of past data

Moving Average and Weighted Moving Average

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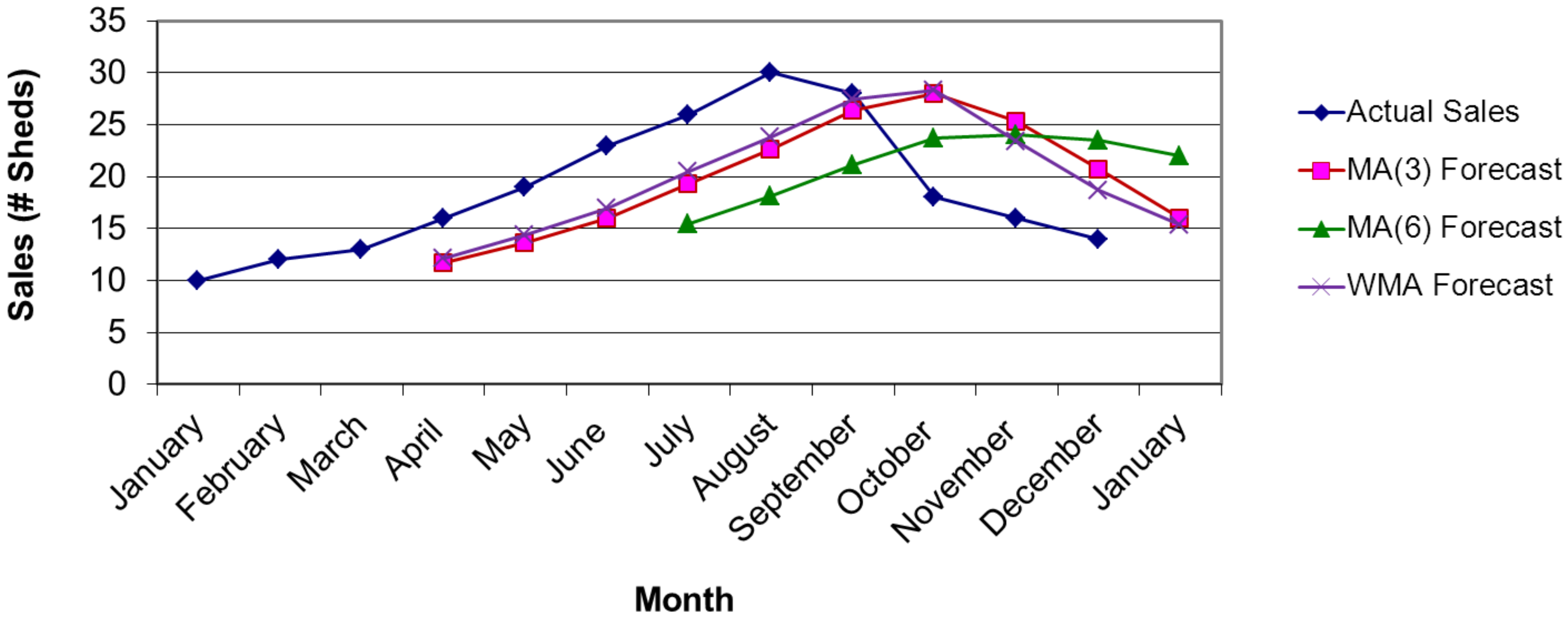

Exponential Smoothing

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- Exponential Smoothing a weighted moving average forecasting technique in which data points are weighted by an exponential function
- The forecast "smooths out" the irregular fluctuations in the time series
- Note: If starting forecast is not given, assume $F_1=A_1$

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$$

Where:

F_t	= New forecast for period t
F_{t-1}	= Previous period's forecast
A_{t-1}	= Previous period's actual demand
α	= ("alpha") the smoothing constant ($0 \le \alpha \le 1$)
(A – F)	= the error term (Actual – Forecast)

Effect of the Smoothing Constant

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$0.0 \le \alpha \le 1.0$

If $\alpha = 0$: $F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$

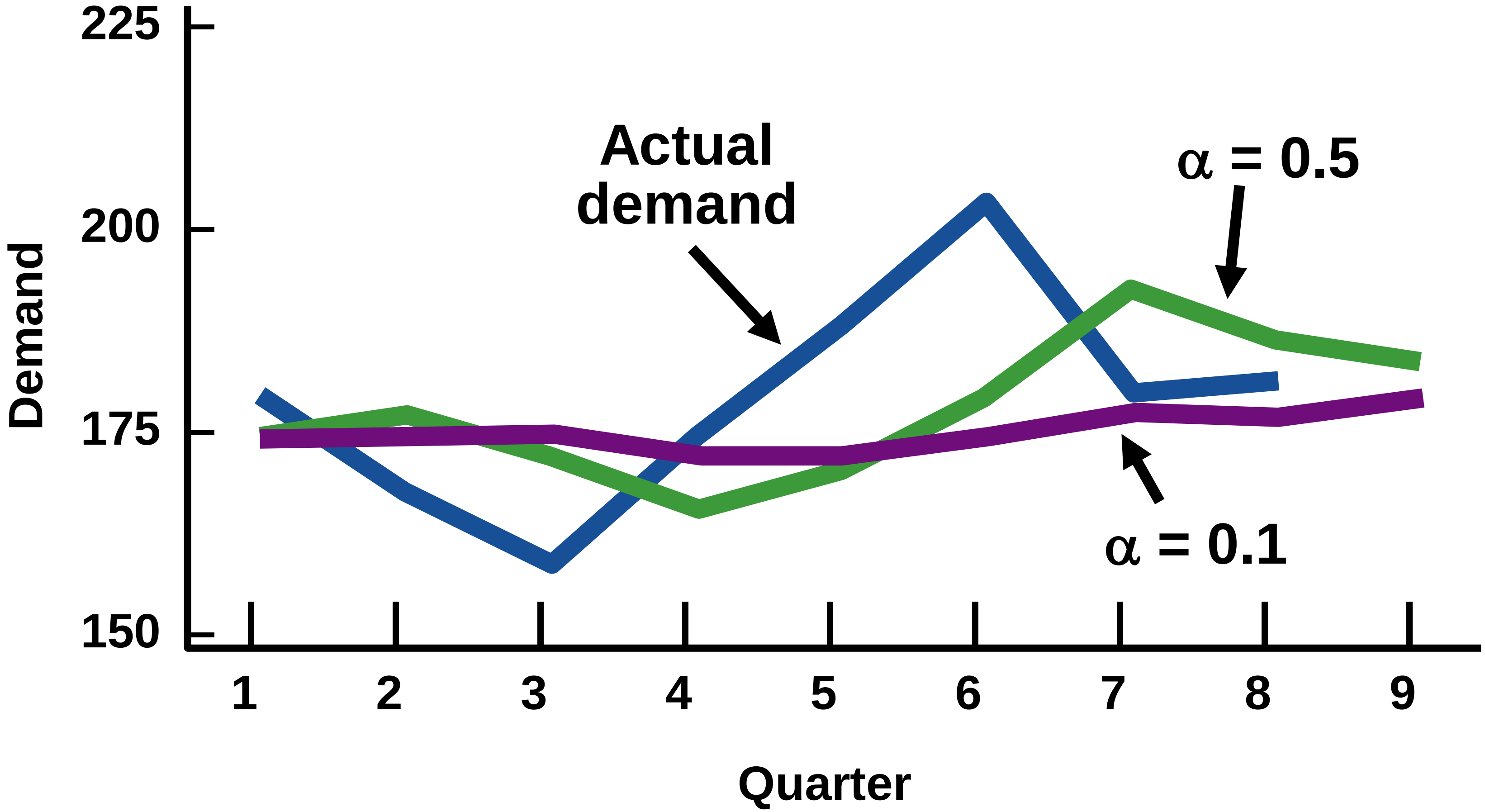
- $F_t = F_{t-1} + 0$
- The forecast does not reflect recent data

If $\alpha = 1$: $F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$

- $F_t = F_{t-1} + 1(A_{t-1} - F_{t-1})$
- $F_t = A_{t-1}$

- Choosing the smoothing constant, α:
  - The closer its value to zero, the *slower* the forecast will be to adjust to forecast errors (the greater the smoothing)
  - The closer its value to one, the greater the responsiveness and the less the smoothing
  - The objective is to obtain the most *accurate* forecast no matter the technique
  - We generally do this by selecting the model that gives us the *lowest forecast error*

Impact of Different α



Exponential Smoothing Example

- In January, a car dealer predicted February demand for 142 Ford Mustangs. Actual February demand was 153 cars. Using a smoothing constant chosen by management of α = 0.20, the dealer want to forecast March demand using the exponential smoothing model.

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$$

Previous forecast = F(t-1) = F(Feb) = 142
Previous Actual demand = A(t-1) = A(Feb) = 153
Smoothing constant α = 0.20

March Fcst = F(Feb) + α[A(Feb) – F(Feb)]
= 142 + 0.2[153 – 142]
= 142 + 2.2
= 144.2 ≈ 144 cars

Measuring Forecast Error

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- Forecast error is the difference between the observed value of the time series and the forecast, or $A_t – F_t$

$$\text{Forecast error} = \text{Actual demand} - \text{Forecast value} = A_t - F_t$$

- Mean Absolute Deviation Error (MAD) - how much the forecast missed the target:

$$\text{MAD} = \frac{\Sigma \left| \text{Actual} - \text{Forecast} \right|}{n}$$

- Mean Square Error (MSE) – the square of how much the forecast missed the target:

$$\text{MSE} = \frac{\Sigma (\text{Forecast errors})^2}{n}$$

- Mean Absolute Percentage Error (MAPE) - the average percent error:

$$\text{MAPE} = \frac{\sum_{i=1}^{n} 100 \left| \text{Actual}_i - \text{Forecast}_i \right| / \text{Actual}_i}{n}$$

Forecast Errors and Accuracy

- A major difference between MSE and MAD is that MSE is influenced much more by *large* forecast errors than by *small* errors (because the errors are squared)
- MAPE is different in that the measurement scale factor is eliminated by dividing the absolute error by the time-series data value. This makes the measure easier to interpret
- The selection of the best measure of forecast accuracy is not a simple matter
  - Forecasting experts often disagree on which measure should be used

Forecast Error Solved Problem

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- Develop:
  1. Three-period moving average forecast
  2. Four-period moving-average forecast
  3. Exponential smoothing forecast with $\alpha = 0.5$
- Compute the MAD, MAPE, and MSE for each of the three forecasts
- Which method provides a better forecast?

Period	Sales (A)
1	86
2	93
3	88
4	89
5	92
6	94
7	91
8	93
9	96
10	97
11	93
12	95

Forecast Error Solved Problem: MA(3)

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Period	Sales (A)	MA(3)
1	86	
2	93	
3	88	
4	89	89.00
5	92	90.00
6	94	89.67
7	91	91.67
8	93	92.33
9	96	92.67
10	97	93.33
11	93	95.33
12	95	95.33
	n =	9

Calculate forecast:

F(4) = [A(3)+A(2)+A(1)]/3

F(4) = [88+93+86]/3 = 89.0

F(5) = [A(4)+A(3)+A(2)]/3

F(5) = [89+88+93]/3 = 90.0

F(12) = [A(11)+A(10)+A(9)]/3

F(12) = [93+97+96]/3 = 95.33

Note that there are a total of 9 forecasts for this data (n = 9)

$$\text{MSE} = \frac{\Sigma(\text{Forecast errors})^2}{n} \qquad \text{MAD} = \frac{\Sigma|\text{Actual - Forecast}|}{n} \qquad \text{MAPE} = \frac{\sum_{i=1}^{n} 100|\text{Actual}_i - \text{Forecast}_i| / \text{Actual}_i}{n}$$

Forecast Error Solved Problem: MA(3)

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Period	Sales (A)	MA(3)	Error
1	86		
2	93		
3	88		
4	89	89.00	0.00
5	92	90.00	2.00
6	94	89.67	4.33
7	91	91.67	-0.67
8	93	92.33	0.67
9	96	92.67	3.33
10	97	93.33	3.67
11	93	95.33	-2.33
12	95	95.33	-0.33
	n =	9	

Calculate forecast errors:

Error = A – F

Error(4) = [A(4) – F(4)]

Error(4) = [89.0 – 89.0] = 0.0

Error(12) = [A(12) – F(12)]

Error(12) = [95.0 – 95.33] = -0.33

$$\text{MSE} = \frac{\Sigma(\text{Forecast errors})^2}{n}$$

$$\text{MAD} = \frac{\Sigma\left|\text{Actual - Forecast}\right|}{n}$$

$$\text{MAPE} = \frac{\sum_{i=1}^{n} 100\left|\text{Actual}_i \text{ - Forecast}_i\right| / \text{Actual}_i}{n}$$

Forecast Error Solved Problem: MA(3)

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Period	Sales (A)	MA(3)	Error	Error ^2
1	86			
2	93			
3	88			
4	89	89.00	0.00	0.00
5	92	90.00	2.00	4.00
6	94	89.67	4.33	18.78
7	91	91.67	-0.67	0.44
8	93	92.33	0.67	0.44
9	96	92.67	3.33	11.11
10	97	93.33	3.67	13.44
11	93	95.33	-2.33	5.44
12	95	95.33	-0.33	0.11
	n =	9		53.78
				5.98
				MSE

Calculate error^2 and use to determine MSE:

Error^2(4) = 0.0^2 = 0.0

Error^2(5) = 2.0^2 = 4.0

Error^2(12) = $(-0.33)^2$ = 0.11

Σ(errors)^2 = Error^2(4) + Error^2(5) +…. Error^2(12)

Σ(errors)^2 = 0.0 + 4.0 +…+ 0.11

Σ(errors)^2 = 53.78

MSE = [Σ(errors)^2] / n

MSE = 53.78 / 9 = 5.98

$$\text{MSE} = \frac{\Sigma(\text{Forecast errors})^2}{n}$$

$$\text{MAD} = \frac{\Sigma\left|\text{Actual - Forecast}\right|}{n}$$

$$\text{MAPE} = \frac{\sum_{i=1}^{n} 100\left|\text{Actual}_i\text{ - Forecast}_i\right| / \text{Actual}_i}{n}$$

Forecast Error Solved Problem: MA(3)

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Period	Sales (A)	MA(3)	Error	Error ^2	Abs Error
1	86				
2	93				
3	88				
4	89	89.00	0.00	0.00	0.00
5	92	90.00	2.00	4.00	2.00
6	94	89.67	4.33	18.78	4.33
7	91	91.67	-0.67	0.44	0.67
8	93	92.33	0.67	0.44	0.67
9	96	92.67	3.33	11.11	3.33
10	97	93.33	3.67	13.44	3.67
11	93	95.33	-2.33	5.44	2.33
12	95	95.33	-0.33	0.11	0.33
	n =	9		53.78	17.33
				5.98	1.93
				MSE	MAD

Calculate lErrorl [=Abs(Error)] and use to determine MAD:

AbsError(4) = Abs(0.0) = 0.0

AbsError(5) =Abs(2.0) = 2.0

AbsError(12) = Abs(-0.33) = 0.33

Σ(Abs Errors) = AbsError(4) + AbsError (5) +…. AbsError (12)

Σ(AbsErrors) = 0.0 + 2.0 +…+ 0.33

Σ(AbsErrors) = 17.33

MAD = [Σ(AbsErrors)] / n

MAD = 17.33 / 9 = 1.93

$$MSE = \frac{\Sigma(\text{Forecast errors})^2}{n}$$

$$MAD = \frac{\Sigma|\text{Actual - Forecast}|}{n}$$

$$MAPE = \frac{\sum_{i=1}^{n} 100|\text{Actual}_i - \text{Forecast}_i| / \text{Actual}_i}{n}$$

Forecast Error Solved Problem: MA(3)

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Period	Sales (A)	MA(3)	Error	Error ^2	Abs Error	(Abs Error/A)*100
1	86					
2	93					
3	88					
4	89	89.00	0.00	0.00	0.00	0.00
5	92	90.00	2.00	4.00	2.00	2.17
6	94	89.67	4.33	18.78	4.33	4.61
7	91	91.67	-0.67	0.44	0.67	0.74
8	93	92.33	0.67	0.44	0.67	0.72
9	96	92.67	3.33	11.11	3.33	3.47
10	97	93.33	3.67	13.44	3.67	3.78
11	93	95.33	-2.33	5.44	2.33	2.51
12	95	95.33	-0.33	0.11	0.33	0.35
	n =	9		53.78	17.33	18.34
				5.98	1.93	2.04
				MSE	MAD	MAPE

Calculate [Abs(Error)/A] *100 and use to determine MAPE:

[AbsError(4)/A(4)]*100 =
= [0.0/89]*100 = 0.0

[AbsError(12)/A(12)]*100
= [0.33/95]*100 = 0.35

Σ[Abs Errors/A]*100 =
[AbsError(4)/A(4)]*100 + ….
[AbsError (12)/A(12)]*100

Σ[AbsErrors/A]*100 = 0.0 +
2.17 +…+ 0.35 = 18.34

MAPE= Σ(AbsErrors/A)*100 / n

MAPE = 18.34 / 9 = 2.04

$$\text{MSE} = \frac{\Sigma(\text{Forecast errors})^2}{n}$$

$$\text{MAD} = \frac{\Sigma|\text{Actual - Forecast}|}{n}$$

$$\text{MAPE} = \frac{\sum_{i=1}^{n} 100|\text{Actual}_i - \text{Forecast}_i| / \text{Actual}_i}{n}$$

Forecast Error Solved Problem

- Develop:
  1. Three-period moving average forecast
  2. Four-period moving-average forecast
  3. Exponential smoothing forecast with α = 0.5
- Compute the MAD, MAPE, and MSE for each of the three forecasts
- Which method provides a better forecast?

Period	Sales (A)
1	86
2	93
3	88
4	89
5	92
6	94
7	91
8	93
9	96
10	97
11	93
12	95

Forecast Error Solved Problem: MA(4)

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Period	Sales (A)	MA(4)	Error	Error ^2	Abs Error	(Abs Error/A)*100
1	86					
2	93					
3	88					
4	89					
5	92	89.00	3.00	9.00	3.00	3.26
6	94	90.50	3.50	12.25	3.50	3.72
7	91	90.75	0.25	0.06	0.25	0.27
8	93	91.50	1.50	2.25	1.50	1.61
9	96	92.50	3.50	12.25	3.50	3.65
10	97	93.50	3.50	12.25	3.50	3.61
11	93	94.25	-1.25	1.56	1.25	1.34
12	95	94.75	0.25	0.06	0.25	0.26
	n =	8		49.69	16.75	17.73
				6.21	2.09	2.22
				MSE	MAD	MAPE

Note that there are a total of 8 forecasts for this data (n = 8)

You should be able to calculate MSE, MAD and MAPE, similar to MA(3)

$$\text{MSE} = \frac{\Sigma(\text{Forecast errors})^2}{n}$$

$$\text{MAD} = \frac{\Sigma\left|\text{Actual - Forecast}\right|}{n}$$

$$\text{MAPE} = \frac{\sum_{i=1} 100\left|\text{Actual}_i - \text{Forecast}_i\right| / \text{Actual}_i}{n}$$

Forecast Error Solved Problem

Operations & Supply Chain University

- Develop:
  1. Three-period moving average forecast
  2. Four-period moving-average forecast
  3. Exponential smoothing forecast with $\alpha = 0.5$
- Compute the MAD, MAPE, and MSE for each of the three forecasts
- Which method provides a better forecast?

Period	Sales (A)
1	86
2	93
3	88
4	89
5	92
6	94
7	91
8	93
9	96
10	97
11	93
12	95

Forecast Error Solved Problem: Exponential Smoothing (α = 0.5)

Operations & Supply Chain University

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$$

Period	Sales (A)	ES (0.5)
1	86	86.00
2	93	86.00
3	88	89.50
4	89	88.75
5	92	88.88
6	94	90.44
7	91	92.22
8	93	91.61
9	96	92.30
10	97	94.15
11	93	95.58
12	95	94.29
	n =	12

Note: If starting forecast is not given, assume F1=A1

F(1) = A(1) since no starting forecast given

F(1) = 86.0

F(2) = F(1) + 0.5[A(1) – F(1)]

F(2) = 86.0 + 0.5[86.0 – 86.0] = 86.0

F(3) = F(2) + 0.5[A(2) – F(2)]

F(3) = 86.0 + 0.5[93.0 – 86.0] = 89.5

F(12) = F(11) + 0.5[A(11) – F(11)

F(12) = 95.58 + 0.5[93.0 - 95.58]

F(12) = 94.29

$$MSE = \frac{\Sigma(\text{Forecast errors})^2}{n}$$

$$MAD = \frac{\Sigma|\text{Actual - Forecast}|}{n}$$

$$MAPE = \frac{\sum_{i=1}^{n} 100|\text{Actual}_i \text{ - Forecast}_i| / \text{Actual}_i}{n}$$

Forecast Error Solved Problem: Exponential Smoothing (α = 0.5)

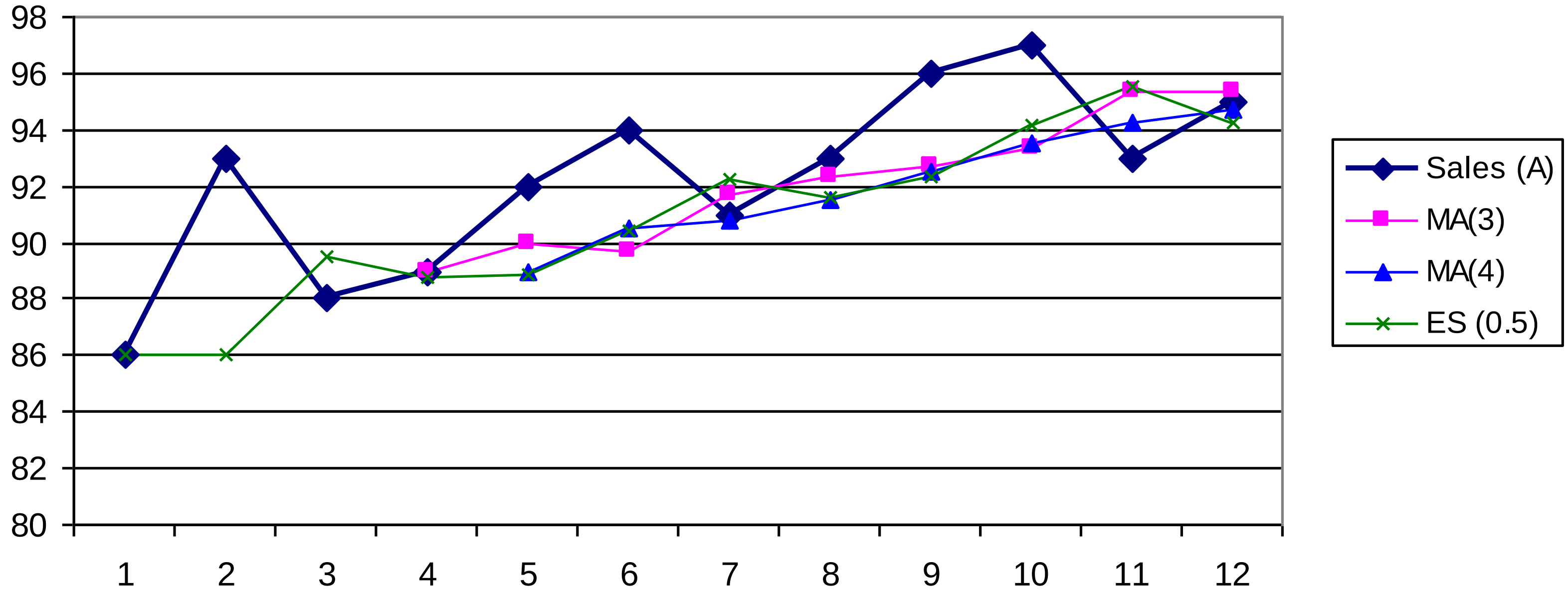
Period	Sales (A)	ES (0.5)	Error	Error ^2	Abs Error	(Abs Error/A)*100
1	86	86.00	0.00	0.00	0.00	0.00
2	93	86.00	7.00	49.00	7.00	7.53
3	88	89.50	-1.50	2.25	1.50	1.70
4	89	88.75	0.25	0.06	0.25	0.28
5	92	88.88	3.13	9.77	3.13	3.40
6	94	90.44	3.56	12.69	3.56	3.79
7	91	92.22	-1.22	1.49	1.22	1.34
8	93	91.61	1.39	1.93	1.39	1.49
9	96	92.30	3.70	13.66	3.70	3.85
10	97	94.15	2.85	8.11	2.85	2.94
11	93	95.58	-2.58	6.64	2.58	2.77
12	95	94.29	0.71	0.51	0.71	0.75
	n =	12		106.10	27.89	29.85
				8.84	2.32	2.49
				MSE	MAD	MAPE

Note that there are a total of 12 Forecasts for this data (n = 12)

You should be able to calculate MSE, MAD and MAPE, similar to MA(3) and MA(4)

$$MSE = \frac{\Sigma(\text{Forecast errors})^2}{n} \qquad MAD = \frac{\Sigma|\text{Actual} - \text{Forecast}|}{n} \qquad MAPE = \frac{\sum_{i=1} 100|\text{Actual}_i - \text{Forecast}_i| / \text{Actual}_i}{n}$$

Forecast Error Solved Problem: Results



	MSE	MAD	MAPE
MA(3)	5.98	1.93	2.04
MA(4)	6.21	2.09	2.22
ES (0.5)	8.84	2.32	2.49

Conclusion: Based on these error metrics (MAD, MSE, MAPE), the 3-month moving average is the best method among the three for ***this data set***

Trend Projections

Operations & Supply Chain University

- Trend projection: a time-series forecasting method that fits a trend line to a series of historical data points and then *projects* the line into the future for forecasts

- Linear trends can be found using the least squares technique

$$\hat{y} = a + bx$$

where $\hat{y}$ = ("y hat") computed value of the variable to be predicted (dependent variable)
a = y-axis intercept
b = slope of the regression line
x = the independent variable (in this case is time)

Least Squares Method

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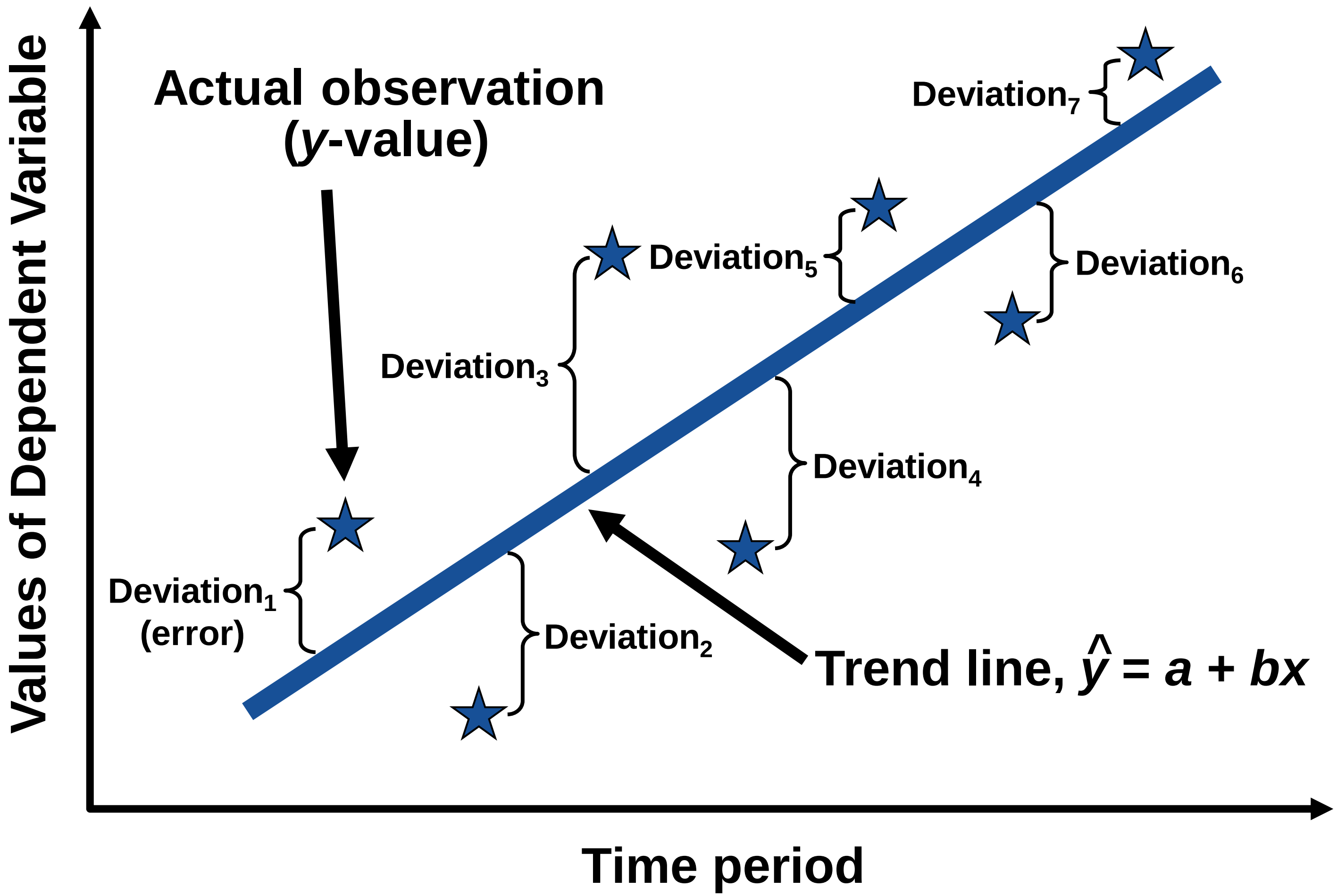


Trend Projections

- Statisticians have developed equations that we can use to find the values of a and b for any regression line
- The slope (b) and the y intercept (a) can be calculated with the following equations:

$$b = \frac{(\Sigma xy) - (n)(\bar{x})(\bar{y})}{(\Sigma x^2) - (n)(\bar{x})^2} \qquad a = \bar{y} - (b)\bar{x}$$

where *b* = slope of the regression line
x = known values of the independent variable
y = known values of the dependent variable
$\bar{x}$ = ("x-bar") average of the x-values
$\bar{y}$ = ("y-bar") average of the y-values
n = number of data points or observations
a = *y*-axis intercept

- We can express the line with the equation: $\hat{y} = a + bx$

Least Squares Example

Operations & Supply Chain University

- The demand for electric power at N.Y. Edison over the past 7 years is shown in the following table, in megawatts. The firm wants to forecast next year's demand by fitting a straight-line trend to these data.

Year	Electrical Power Demand (megawatts)
1	74
2	79
3	80
4	90
5	105
6	142
7	122

Least Squares Example

Operations & Supply Chain University

Year	Time Period (x)	Electrical Power Demand (y)
1	1	74
2	2	79
3	3	80
4	4	90
5	5	105
6	6	142
7	7	122
n=7	$\sum x = 28$ $\bar{x} = 28/7$ $\bar{x} = 4$	$\sum y = 692$ $\bar{y} = 692/7$ $\bar{y} = 98.86$

Least Squares Example

Operations & Supply Chain University

Year	Time Period (x)	Electrical Power Demand (y)	x^2
1	1	74	1
2	2	79	4
3	3	80	9
4	4	90	16
5	5	105	25
6	6	142	36
7	7	122	49
n=7	$\sum x = 28$	$\sum y = 692$	$\sum x^2 = 140$
	$\bar{x} = 4$	$\bar{y} = 98.86$	

Least Squares Example

Operations & Supply Chain University

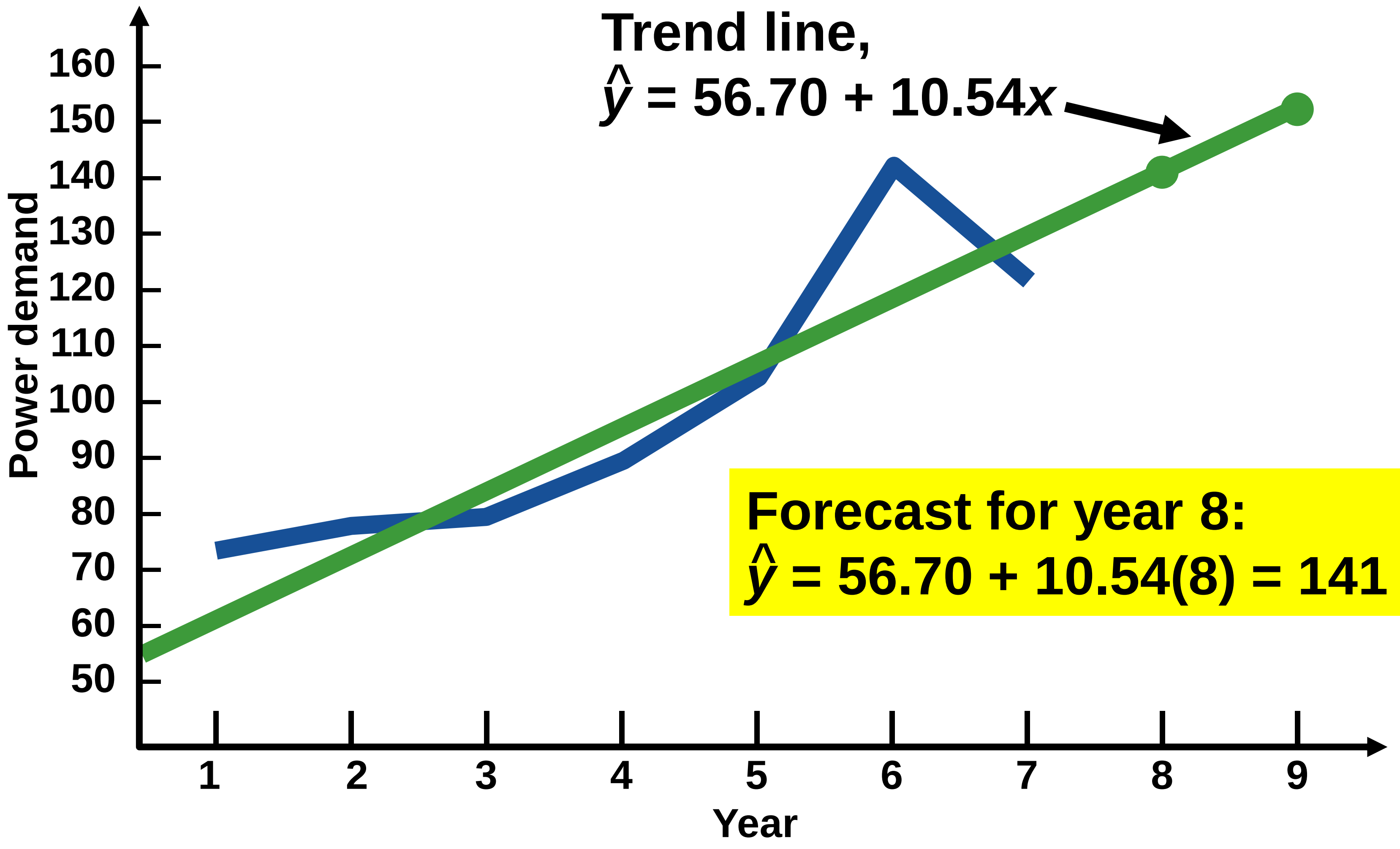
Year	Time Period (x)	Electrical Power Demand (y)	x^2	xy
1	1	74	1	74
2	2	79	4	158
3	3	80	9	240
4	4	90	16	360
5	5	105	25	525
6	6	142	36	852
7	7	122	49	854
n=7	$\sum x = 28$	$\sum y = 692$	$\sum x^2 = 140$	$\sum xy = 3{,}063$
	$\bar{x} = 4$	$\bar{y} = 98.86$		

$$b = \frac{(\sum xy) - (n)(\bar{x})(\bar{y})}{(\sum x^2) - n(\bar{x})^2} = \frac{3{,}063 - (7)(4)(98.86)}{140 - (7)(4^2)} = 10.54$$

$$a = \bar{y} - b\bar{x} = 98.86 - 10.54(4) = 56.70$$

$$\text{Trend line} = \hat{y} = a + bx = 56.70 + 10.54x$$

Least Squares Example



Least Squares Requirements

- Always plot the data to insure a linear relationship
- Do not predict time periods far beyond the database
- Deviations around the least squares line are assumed to be *random*

Regression Analysis

- Regression analysis: a straight-line mathematical model to describe the functional relationships between independent and dependent variables
  - The dependent variable will still be y but the independent variable (x) need no longer be *time*

$$\hat{y} = a + bx$$

where $\hat{y}$ = ("y hat") computed value of the variable to be predicted (dependent variable)
a = y-axis intercept
b = slope of the regression line
x = the independent variable (not time)

- MS Excel provides a very simple tool to find the best-fitting regression model for a time series by selecting the "Add Trend line" option

Linear Regression Forecasting Example

Operations & Supply Chain University

Nodel Construction Company renovates old homes in West Bloomfield, Michigan. Over time, the company has found that its dollar volume of renovation work is dependent on the West Bloomfield area payroll. Management wants to establish a mathematical relation to help predict sales.

Area Payroll (in $ billions), x	Nodel's Sales (in $ millions), y
1.0	2.0
3.0	3.0
4.0	2.5
2.0	2.0
1.0	2.0
7.0	3.5

Steps:

1. Calculate: **n, Σx, Σy, Σxy, Σx^2**
2. Substitute into equations to solve for a and b
3. Substitute a and b into regression equation
4. Use regression equation to forecast

Linear Regression Forecasting Solution

Operations & Supply Chain University

Steps:

1. **Calculate: n, Σx, Σy, Σxy, Σx^2**
2. **Substitute into equations to solve for a and b**
3. **Substitute a and b into regression equation**
4. **Use regression equation to forecast**

n	Area Payroll (in $ billions), x	Nodel's Sales (in $ millions), y	x^2	xy
1	1.0	2.0	1.0	2.0
2	3.0	3.0	9.0	9.0
3	4.0	2.5	16.0	10.0
4	2.0	2.0	4.0	4.0
5	1.0	2.0	1.0	2.0
6	7.0	3.5	49.0	24.5
n=6	Σx = 18.0	Σy= 15.0	Σx^2 = 80	Σxy = 51.5

$\bar{x} = \Sigma x/n = 18.0/6 = 3.0$, $\bar{y} = \Sigma y/n = 15.0/6 = 2.5$

$$b = \frac{(\Sigma xy) - (n)(\bar{x})(\bar{y})}{(\Sigma x^2) - (n)(\bar{x})^2} = \frac{51.5 - 6(3)(2.5)}{80 - 6(3)^2}$$

b = 0.25

$a = \bar{y} - b\bar{x} = 2.5 - (0.25)(3)$

a = 1.75

So the linear regression equation is:

$\hat{y} = a + bx = 1.75 + 0.25x$

Or: Sales = 1.75 + 0.25 (payroll)

If the Chamber of Commerce predicts payroll to be $6B, estimated sales:

Sales (millions) = 1.75 + 0.25 (6) = 1.75 + 1.50 = 3.25 or $3,250,000

Linear Regression Example Using Excel

Steps to use Excel:

1. Open new spreadsheet and input x and y data
   - Insert: Chart
   - Select chart type: Scatter
   - Data indicates fairly linear relationship

2. "Select" the plotted data, right click and select "Add Trendline"
   - Select: "Linear"
   - Select Options: Display equation and R-squared value

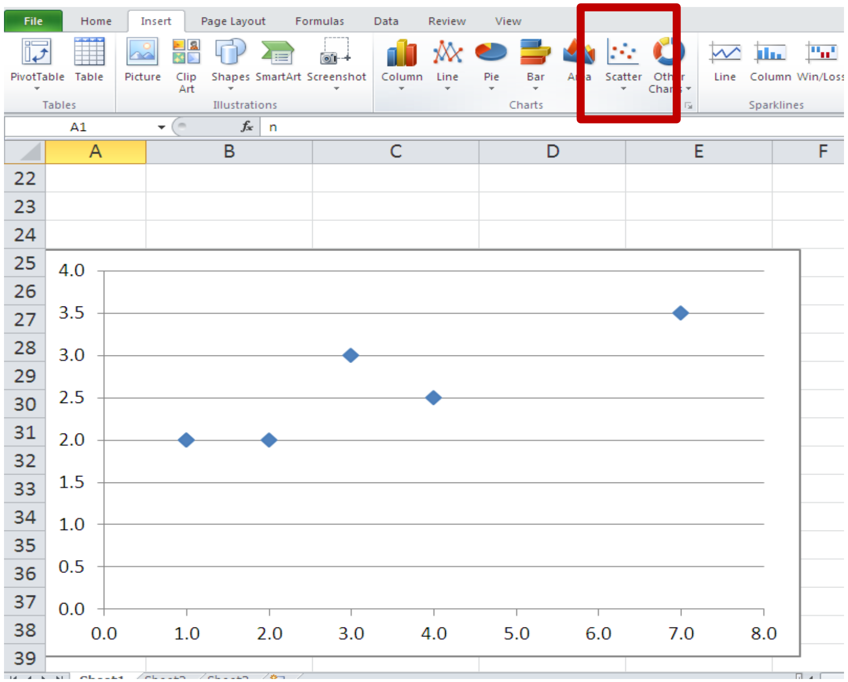
3. Calculate forecast and evaluate correlation

Plot Data and Create Scatter Diagram

Operations & Supply Chain University

1. Open new spreadsheet and input x and y data
   - Insert: Chart
   - Select chart type: Scatter
   - Data indicates fairly linear relationship ‘
     - If it is not linear, should not use this method

Payroll, x	Sales, y
1.0	2.0
3.0	3.0
4.0	2.5
2.0	2.0
1.0	2.0
7.0	3.5



Add Trendline and Display Output

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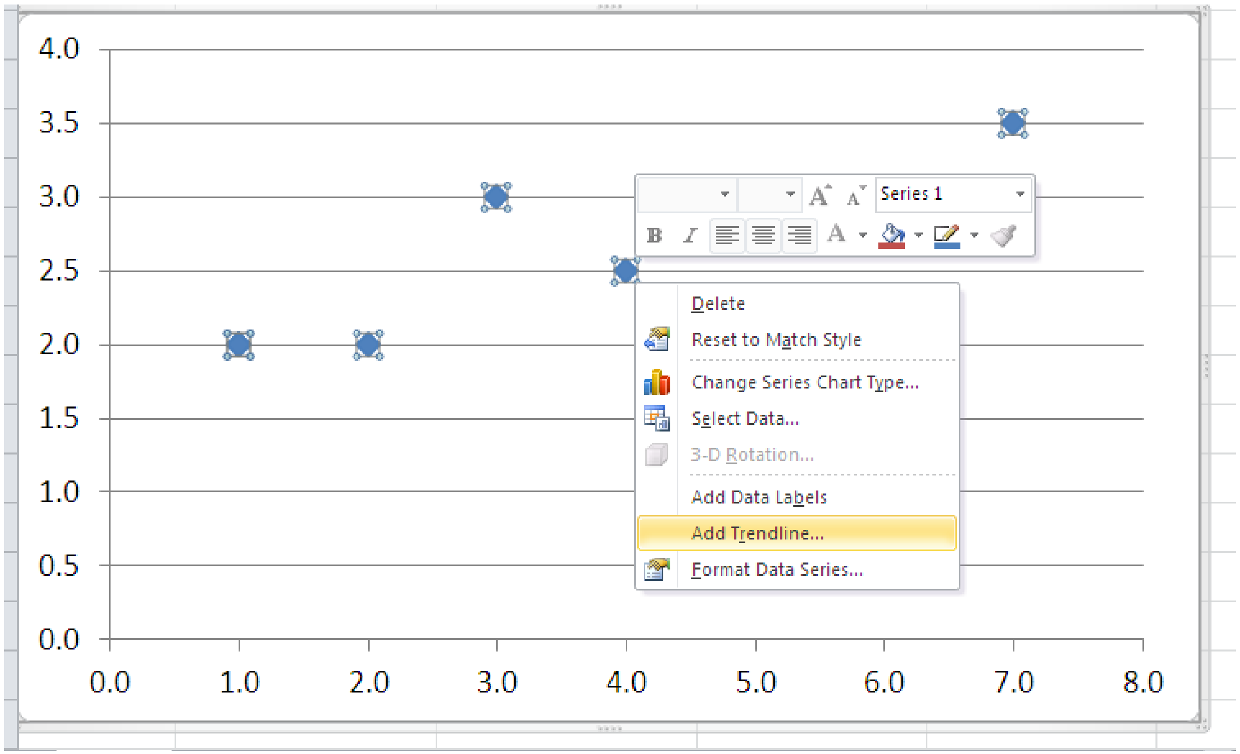
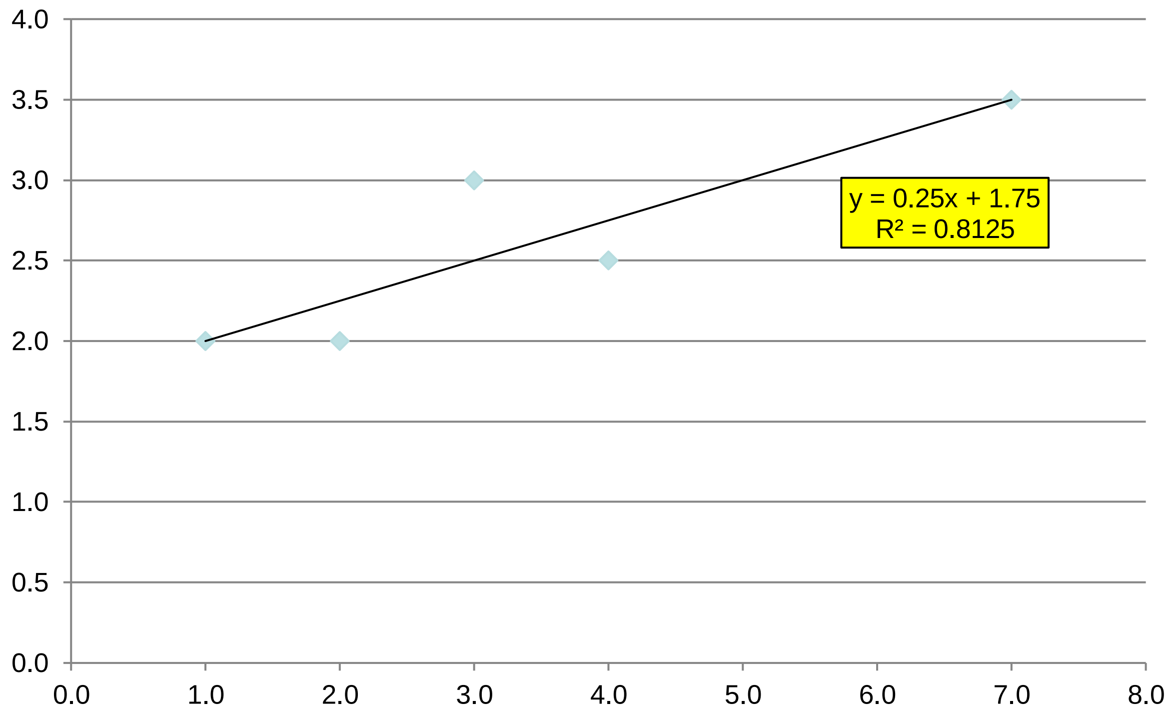
2. "Select" the plotted data, right click and select "Add Trendline"
    - Select: "Linear"
    - Select Options: Display equation and R-squared value



Calculate Forecast and Evaluate Correlation

- Result: y = 0.25x + 1.75, R^2 = 0.8125
- If payroll is predicted to be $6B, estimated sales:

  y = 0.25x + 1.75 → y = 0.25 (6) + 1.75 = $3.25 M



Correlation Coefficient (r) and Coefficient of Determination (r^2)

Operations & Supply Chain University

Correlation Coefficient (r):

- A measure of the **strength** of the linear relationship between **independent** and **dependent** variables
  - Varies between -1.00 and + 1.00

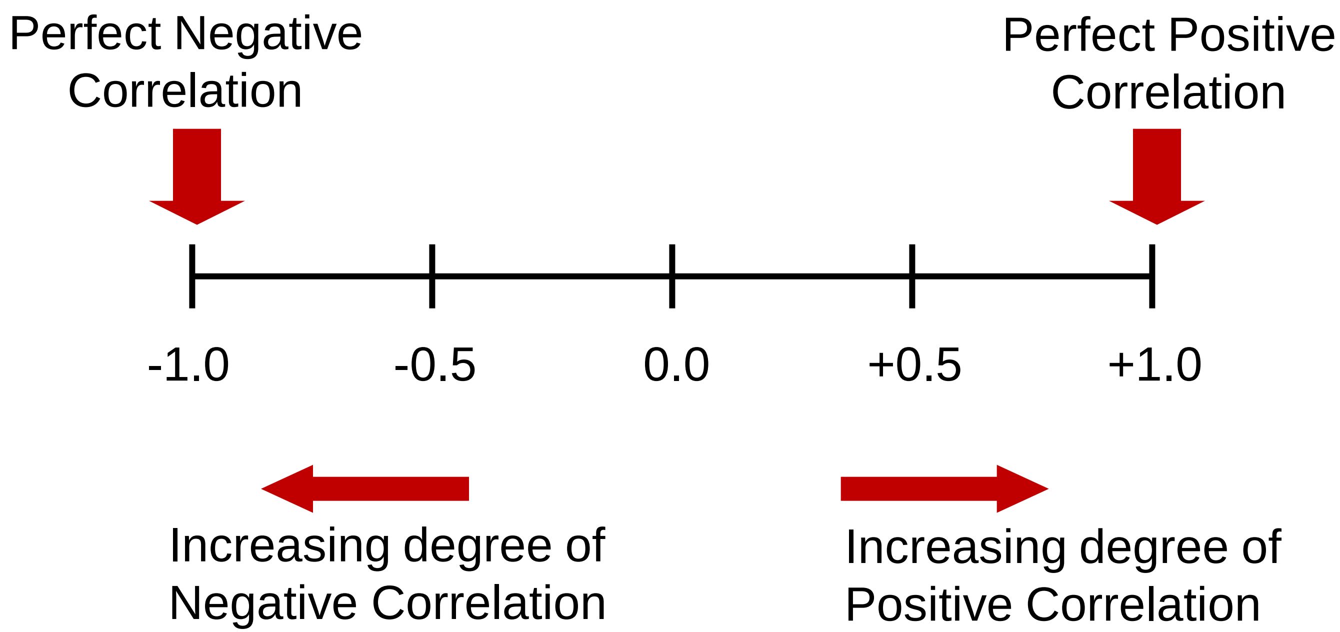
Coefficient of Determination (r^2):

- The **percentage** of the variation in the dependent variable that results from the independent variable
  - Varies between 0 and 1

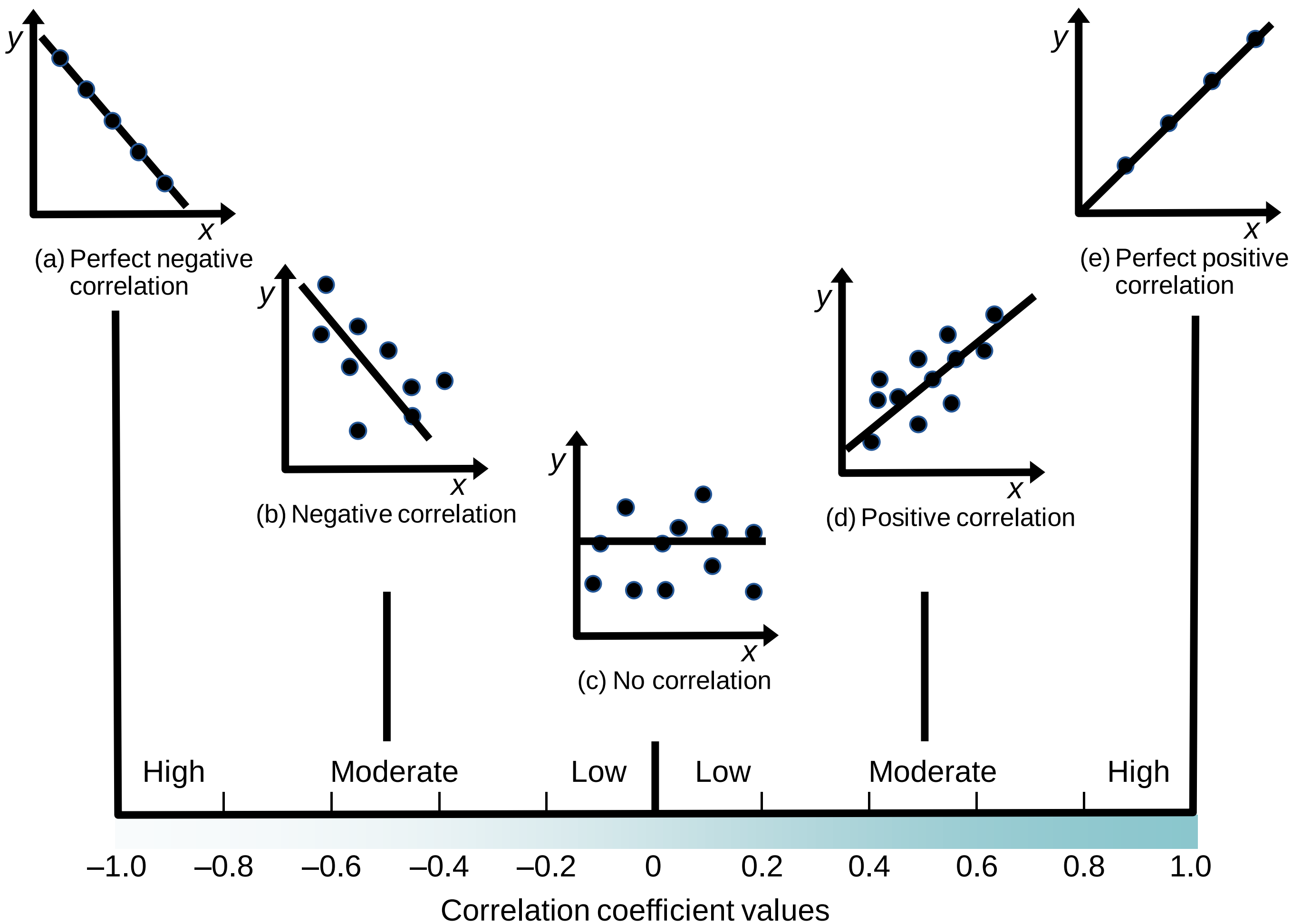
Correlation Coefficient

<u>Correlation Coefficient (r):</u>

- A measure of the strength of the linear relationship between independent and dependent variables
- Varies between -1.00 and + 1.00



Correlation Coefficient (r)



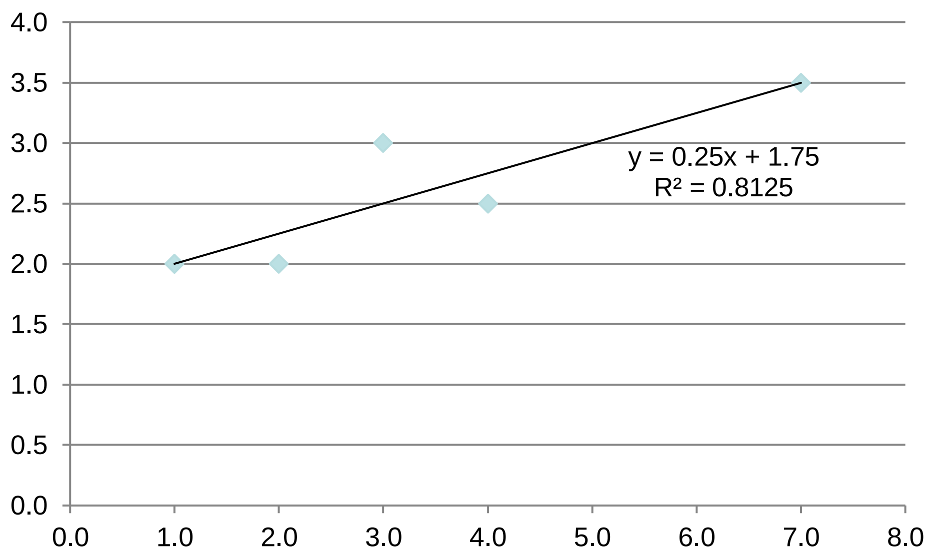
Calculate Forecast and Interpret Correlation

Operations & Supply Chain University

- If payroll is predicted to be $6B, estimated sales:

  $y = 0.25x + 1.75 \rightarrow y = 0.25\,(6) + 1.75 = \$3.25$ M

- $R^2 = 0.8125$ **….what does this mean???**



<u>Coefficient of Determination (r^2):</u>
The percentage of the variation in the dependent variable that results from the independent variable. The r^2 varies between 0 and 1.

With $R^2 = 0.8125$, it indicates a strong dependence

<u>Correlation Coefficient (r):</u>
A measure of the strength of the linear relationship between independent and dependent variables

R = 0.9014…therefore, strong positive correlation

Seasonal Variations In Data

Operations & Supply Chain University

- The multiplicative seasonal model can adjust trend data for seasonal variations in demand
- Steps in process
  1. Find ***average historical demand*** for each season
  2. Compute the ***average demand*** over all seasons
  3. Compute a ***seasonal index*** for each season
  4. Estimate next year's total demand
  5. Divide this estimate of total demand by the number of seasons, then multiply it by the seasonal index for that season →this provides the ***seasonal forecast***

Seasonal Index Example

- A Des Moines distributor of Sony laptop computers wants to develop monthly indices for sales. Data from the past three years are shown below:

Month	Demand		
	Year 1	Year 2	Year 3
Jan	80	85	105
Feb	70	85	85
Mar	80	93	82
Apr	90	95	115
May	113	125	131
Jun	110	115	120
Jul	100	102	113
Aug	88	102	110
Sep	85	90	95
Oct	77	78	85
Nov	75	82	83
Dec	82	78	80

Seasonal Index Example

Operations & Supply Chain University

- A Des Moines distributor of Sony laptop computers wants to develop monthly indices for sales. Data from the past three years are shown below:

Month	Demand			Average
	Year 1	Year 2	Year 3	Yr 1-3
Jan	80	85	105	90
Feb	70	85	85	80
Mar	80	93	82	85
Apr	90	95	115	100
May	113	125	131	123
Jun	110	115	120	115
Jul	100	102	113	105
Aug	88	102	110	100
Sep	85	90	95	90
Oct	77	78	85	80
Nov	75	82	83	80
Dec	82	78	80	80

Jan Average Y1-Y3
= (JanY1 + JanY2 + JanY3)/3
= (80 + 85 + 105)/3 = 90

Feb Average Y1-Y3
= (FebY1+ FebY2 + FebY3)/3
= (70 + 85 + 85)/3 = 80

Average of Monthly Average
= (Jan Y1-Y3 + Feb Y1-Y3 + …
+ Dec Y1-Y3)/12
= (90 + 80 + … + 80)/12
= 1128 / 12 = 94

Seasonal Index Example

Operations & Supply Chain University

- A Des Moines distributor of Sony laptop computers wants to develop monthly indices for sales. Data from the past three years are shown below:

Month	Demand			Average	Average
	Year 1	Year 2	Year 3	Yr 1-3	Monthly
Jan	80	85	105	90	94
Feb	70	85	85	80	94
Mar	80	93	82	85	94
Apr	90	95	115	100	94
May	113	125	131	123	94
Jun	110	115	120	115	94
Jul	100	102	113	105	94
Aug	88	102	110	100	94
Sep	85	90	95	90	94
Oct	77	78	85	80	94
Nov	75	82	83	80	94
Dec	82	78	80	80	94

From prior calculation, average of monthly average = 94 per month

Seasonal Index Example

Operations & Supply Chain University

- A Des Moines distributor of Sony laptop computers wants to develop monthly indices for sales. Data from the past three years are shown below:

Month	Demand			Average	Average	Seasonal
	Year 1	Year 2	Year 3	Yr 1-3	Monthly	Index
Jan	80	85	105	90	94	0.957
Feb	70	85	85	80	94	0.851
Mar	80	93	82	85	94	0.904
Apr	90	95	115	100	94	1.064
May	113	125	131	123	94	1.309
Jun	110	115	120	115	94	1.223
Jul	100	102	113	105	94	1.117
Aug	88	102	110	100	94	1.064
Sep	85	90	95	90	94	0.957
Oct	77	78	85	80	94	0.851
Nov	75	82	83	80	94	0.851
Dec	82	78	80	80	94	0.851

Jan Seasonal Index
= JanAvg/ MonthlyAvg
= 90 / 94 = 0.957

Feb Seasonal Index
= FebAvg/ MonthlyAvg
= 80 / 94 = 0.851

Jul Seasonal Index
= JulAvg/ MonthlyAvg
= 105 / 94 = 1.117

Seasonal Index Example

Operations & Supply Chain University

- If we expected annual demand for next year for computers to be 1150 units, we would use these seasonal indices to forecast the monthly demand (→1150/12=95.83)
- Forecast = Monthly Demand x Seasonal Index

Month	Demand			Average	Average	Seasonal
	Year 1	Year 2	Year 3	Yr 1-3	Monthly	Index
Jan	80	85	105	90	94	0.957
Feb	70	85	85	80	94	0.851
Mar	80	93	82	85	94	0.904
Apr	90	95	115	100	94	1.064
May	113	125	131	123	94	1.309
Jun	110	115	120	115	94	1.223
Jul	100	102	113	105	94	1.117
Aug	88	102	110	100	94	1.064
Sep	85	90	95	90	94	0.957
Oct	77	78	85	80	94	0.851
Nov	75	82	83	80	94	0.851
Dec	82	78	80	80	94	0.851

F(Jan)
=(95.83)(0.957)
=91.7 = 92

F(Feb)
=(95.83)(0.851)
=81.6 = 82

F(Jul)
=(95.83)(1.117)
=107

Seasonal Index Example

Operations & Supply Chain University