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Module B: Linear Programming

Learning Objectives:

- Formulate linear programming models, including an objective function and constraints
- Recognize decision variables, the objective function and constraints in formulating linear optimization models
- Formulate a linear programming model from a description of a problem
- Solve linear programming problems using algebraic methods
- Use Excel Solver to solve linear optimization models on spreadsheets
- Interpret computer solutions of linear programming problems
- Perform sensitivity analysis on the solution of a linear programming problem

Linear Programming

- A model represents the essential features of an object, system or problem without all of the unimportant details
  - A simplified version of reality
  - Mathematical models are cheaper, faster and safer than constructing and manipulating real systems

- Optimization Models: models that seek to maximize or minimize some objective function while satisfying a set of constraints
  - An important category of optimization models is linear programming

- Linear Programming (LP): A mathematical technique designed to help operations managers plan and make decisions necessary to allocate resources

Uses of Linear Programming

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- Many operations management decisions involve trying to make the most effective use of resources – LP problems seek to maximize or minimize some quantity
  - Scheduling school buses to *minimize* total distance traveled when carrying students
  - Allocating police patrol units to high crime areas to *minimize* response time to 911 calls
  - Selecting the product mix in a factory to make best use of machine- and labor-hours available while *maximizing* the firm's profit
  - Picking blends of raw materials in feed mills to produce finished feed combinations at *minimum* costs
  - Allocating space for a tenant mix in a new shopping mall so as to *maximize* revenues for the leasing company

Requirements for Linear Programming

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- There are four essential conditions for Linear Programming:
  1. LP problems seek to maximize or minimize some quantity (usually profit or cost) expressed as an objective function
  2. The presence of restrictions, or constraints, limits the degree to which we can pursue our objective
  3. There must be *alternative* courses of action to choose from
  4. The objective and constraints in linear programming problems must be expressed in terms of *linear* equations or inequalities

- There are other requirements for LP… There must be
  - ✓ Limited resources
  - ✓ An explicit objective
  - ✓ Linearity / Divisibility
  - ✓ Homogeneity

LP Model Components

- There are **three key components** of a Linear Programming model: decision variables, an objective function and constraints

1. Decision variables: controllable input variable that represents the key decisions a manager must make to achieve an objective
   - Generally use x1, x2, x3, etc. to represent decision variables

2. Objective Function: the evaluation criteria (often maximizing profit or minimizing cost)
   - Objective Function Coefficients: The constant terms in the objective function

LP Model Components

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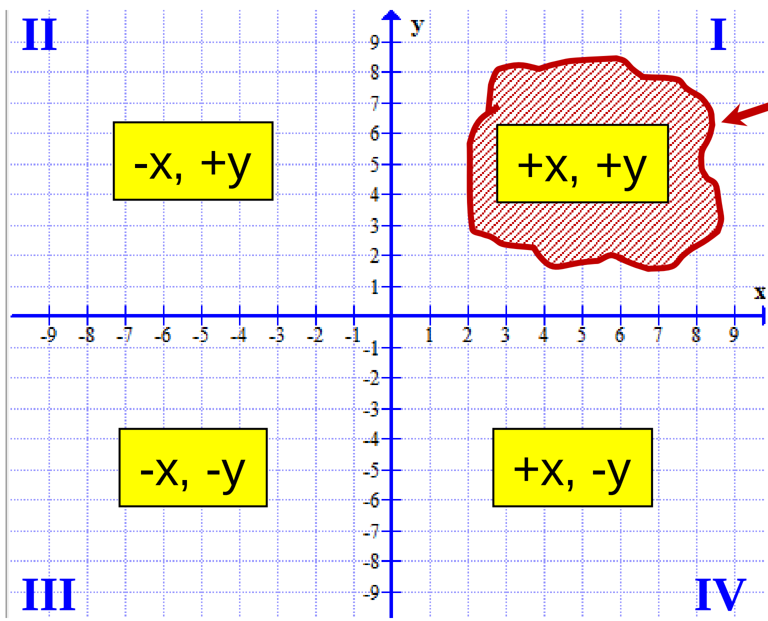
3. Constraint: some *limitation* or requirement that must be satisfied by the solution. There are three types of constraints
   - Upper limits where the amount used is ≤ the amount of a resource (less than or equal too).
   - Lower limits where the amount used is ≥ the amount of the resource (greater than or equal too).
   - Equalities where the amount used is = the amount of the resource

- Solution: Any particular combination of decision variables
  - Feasible Solutions: solutions that satisfy *all* constraints
  - Optimal Solution: any feasible solution that optimizes the objective function

LP Model Components

- Additional LP requirements:
  - The Objective Function and Constraints must be expressed in linear terms of equations or inequalities
  - Values of parameters are known and constant
  - Decision variables must be divisible and non-negative (aka: positive)



Cartesian Coordinate System

The non-negativity requirement ensures all solutions will be in quadrant **I** and will be positive values

LP Mathematical Programming Convention

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- Objective Function
  - Max Z =
  - Min Z =
- Decision variables
  - Must be ≥ 0 (non-negativity constraint)
- Standard notation
  - Z on left-hand side (LHS)
  - Decision variables on right-hand side (RHS)
- Constraints format
  - ≥ or >= (greater than or equal to)
  - ≤ or <= (less than or equal to)
  - =

Formulating and Solving the Model

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- How to formulate the model:
  1. Define in words the objective you are trying to achieve
  2. List the decisions that are to be made
  3. Write the objective function
  4. List the constraining factors that affect these decisions

- Many methods to solve the model:
  - Guessing
  - Graph
  - Algebraic methods
  - Simplex Method
  - Software (Excel/Solver, POM, etc.)

LP Example: Glickman Electronics

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- The Glickman Electronics Company in Washington DC produces two products:
  - The Glickman x-pod, a portable music player
  - The Glickman BlueBerry, an internet connected color telephone
- The production process for each product is similar, there are 240 hours of electronic and 100 hours of assembly time available
  - Each x-pod takes 4 hours of electronic work and 2 hours in the assembly shop
  - Each BlueBerry requires 3 hours in electronics and 1 hour in assembly
- Each x-pod sold yields a profit of $7 and each BlueBerry yields a profit of $5
- Q: What is the best combination of x-pods and BlueBerrys to reach the maximum profit?

Glickman Electronics: Problem Summary

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	Hours Required to Produce One Unit		
Department	**X-Pods (x1)**	**Blueberrys (x2)**	**Available Hours per Week**
Electronic	4	3	240
Assembly	2	1	100
Profit per Unit	$7	$5	

► Q: What is the best combination of x-pods and BlueBerrys to reach the maximum profit?

Glickman Electronics: Decision Variables

- Decision variables: controllable input variable that represents the key decisions a manager must make to achieve an objective
  - Generally use x1, x2, x3, etc. to represent decision variables
- For Glickman Electronics:

Glickman Decision Variables:

x1 = number of x-pods to produce

x2 = number of BlueBerrys to produce

Glickman Electronics: Objective Function

- Objective Function: the evaluation criteria (often maximizing profit or minimizing cost)
- For Glickman Electronics:

Glickman Objective Function:

Maximize profit = $7(number of x-pods) + $5(number of BlueBerrys)

Maximize z = $7x1 + $5x2

↑ ↑

Objective function coefficients (constant terms)

Glickman Electronics: Constraints

- Constraint: some limitation or requirement that must be satisfied by the solution
- For Glickman Electronics:

Glickman Constraints:

Electronic time used is <= electronic time available:

Electronic constraint: 4(x-pods) + 3(BlueBerrys) <= 240

Electronic constraint: 4x1 + 3x2 <= 240

Assembly time used is <= assembly time available:

Assembly constraint: 2(x-pods) + 1(BlueBerrys) <= 100

Assembly constraint: 2x1 + 1x2 <= 100

Non-negativity constraints:

x1 >= 0

x2 >= 0

Glickman Electronics: Mathematical Model

- Decision variables

  $x1$ = number of x-pods to produce

  $x2$ = number of BlueBerrys to produce

- Objective Function: maximize profit

  Max $z = \$7x1 + \$5x2$

- Subject to (constraints):

  Electronic constraint: $4x1 + 3x2 \leq 240$
  Assembly time constraint: $2x1 + 1x2 \leq 100$

  Non-negativity constraints: $x1, x2 \geq 0$

► Our task is now to find the product mix (the combination of x1 and x2) that satisfies all the constraints and, at the same time, yields a value for the objective function that is greater than or equal to the value given by any other feasible solution

Summary of key steps and definitions

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- How to formulate the model:
  1. Define in words the objective you are trying to achieve
  2. List the decisions that are to be made
  3. Write the objective function
  4. List the constraining factors that affect these decisions

- Key definitions:
  1. Decision variables: controllable input variables that represent the decision to be made (ex: how many to produce?)
  2. Objective function: The evaluation criteria (ex: what will max profit?)
  3. Constraints: some limitation or requirements that must be satisfied by the solution (ex: how many labor hours are available?)

Linear Programming Using Microsoft Excel

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- Spreadsheets can be used to solve linear programming problems
- Microsoft Excel has an optimization tool called Solver for this purpose
- You will need to download the Solver Add-In
  - See "Mod B-Downloading Solver.ppt" (posted on BB)

Glickman Electronics: Problem Summary

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	Hours Required to Produce One Unit		
Department	X-Pods (x1)	Blueberrys (x2)	Available Hours per Week
Electronic	4	3	240
Assembly	2	1	100
Profit per Unit	$7	$5	

- Now, let's put this in Excel…

Glickman Excel Spreadsheet Programming

Glickman Electronics: Mathematical Model

- Decision variables
  - **x1 = number of x-pods to produce**
  - **x2 = number of BlueBerrys to produce**
- Objective Function: maximize profit
  - **Max z = $7x1 + $5x2**
- Subject to (constraints):
  - **Electronic constraint: 4x1 + 3x2 ≤ 240**
  - **Assembly time constraint: 2x1 + 1x2 ≤ 100**
  - **Non-negativity constraints: x1, x2 ≥ 0**

	A	B	C	D	E	F
1	Glickman Electronics	Objective Function:	Max z = 7x1 + 5x2			
2						
3		x1 = x-pods	x2 = BlueBerrys	Left-hand side		Right-hand side
4	Objective Function	7	5	=B4*B8+C4*C8		
5	Electronics	4	3	=B5*B8+C5*C8	<=	240
6	Assembly	2	1	=B6*B8+C6*C8	<=	100
7						
8	Solution Values	0	0			

Glickman Spreadsheet Linkage to Excel Solver

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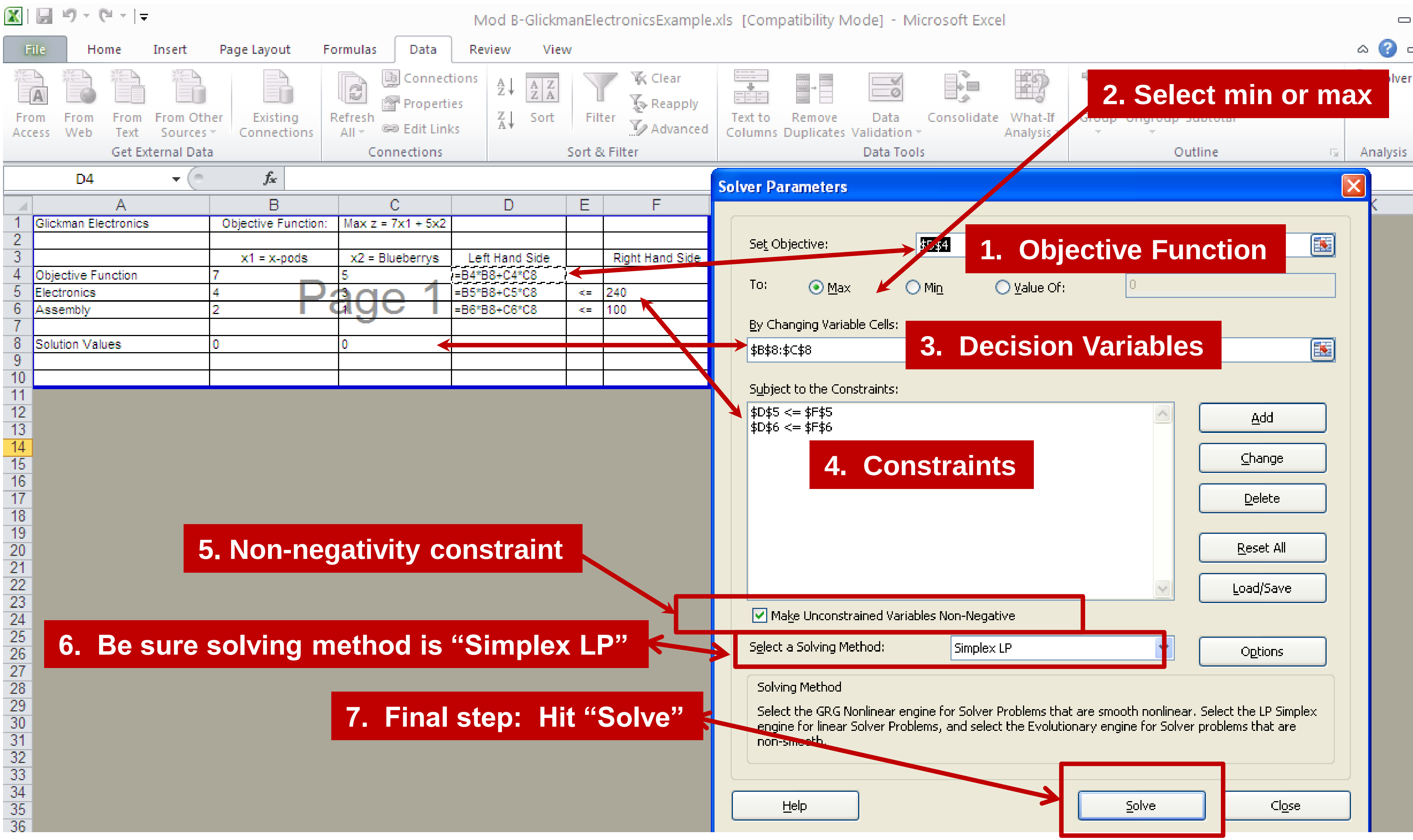

Solver Finds Solution!

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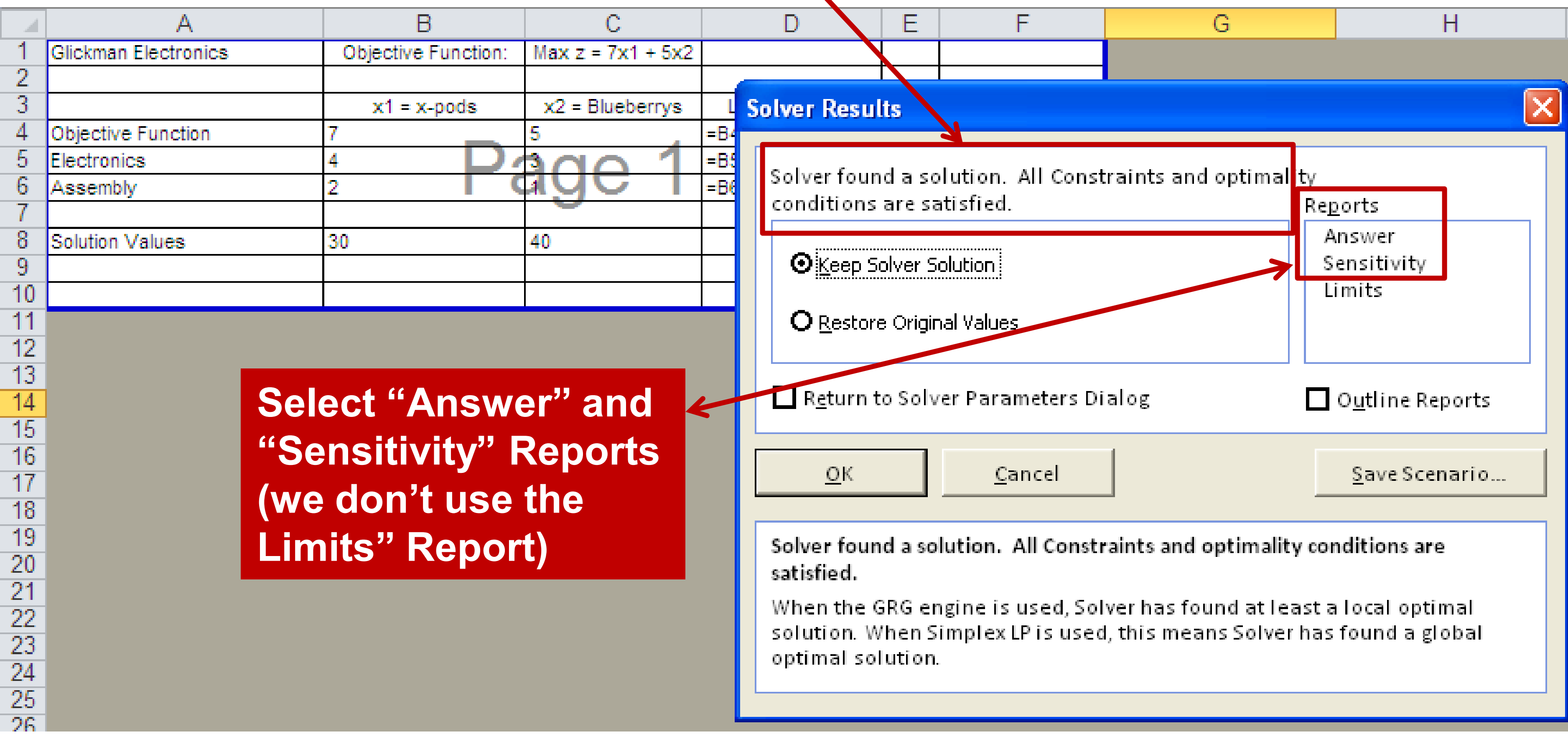
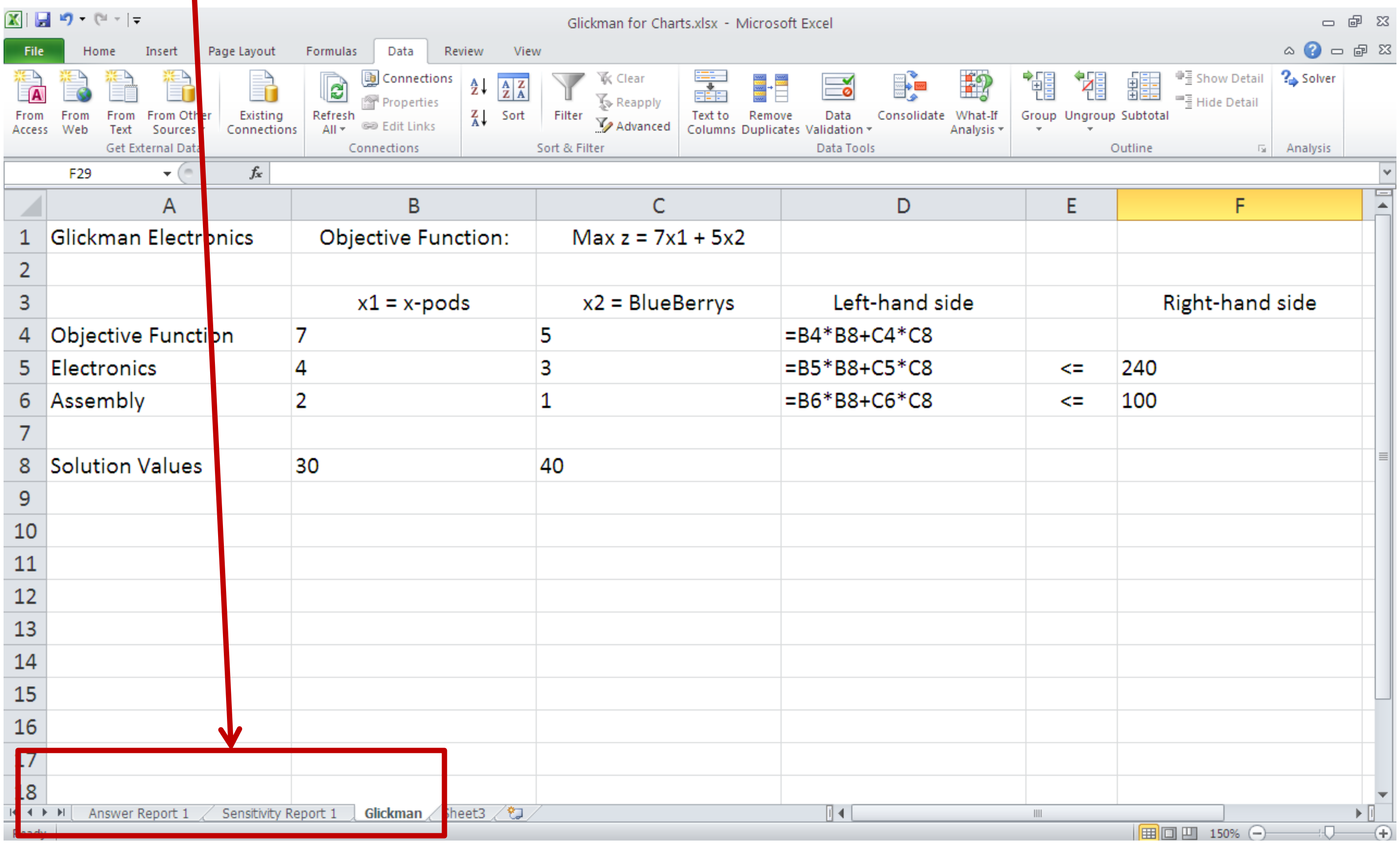
Note: for all HW and Practice Problems, a solution is always possible! If Solver can't find the Solution, YOU made a mistake!



Select "Answer" and "Sensitivity" Reports (we don't use the Limits" Report)

Answer and Sensitivity Reports appear

Answer and Sensitivity Reports appear as new worksheets prior to data sheet



Excel Solver: Answer Report

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Microsoft Excel 14.0 Answer Report

This tells us we will make $410 profit

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$D$4	Objective Function Left Hand Side	0	410

By manufacturing 30 x1's (x-pods) and 40 x2's (BlueBerrys)

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$8	Solution Values x1 = x-pods	0	30	Contin
$C$8	Solution Values x2 = Blueberrys	0	40	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$5	Electronics Left Hand Side	240	$D$5<=$F$5	Binding	0
$D$6	Assembly Left Hand Side	100	$D$6<=$F$6	Binding	0

Excel Solver: Sensitivity Report

Excel Solver: Sensitivity Report
(We will discuss how to use this report soon!)

Microsoft Excel 14.0 Sensitivity Report

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$8	Solution Values x1 = x-pods	30	0	7	3	0.333333333
$C$8	Solution Values x2 = Blueberrys	40	0	5	0.25	1.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$5	Electronics Left Hand Side	240	1.5	240	60	40
$D$6	Assembly Left Hand Side	100	0.5	100	20	20

Graphical Method

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Graphical Method (to find the optimal solution to two-variable problems)

1. Set up objective function and constraints in mathematical format
2. Plot the constraints
   - For first constraint, replace the inequality (>= or <=) with an equal sign (=)
   - Determine where the constraint intersects each axis
     - Assume x1 = 0, solve for x2
     - Assume x2 = 0, solve for x1
     - Plot the line represented by the points above
   - Repeat for each constraint
3. Identify the area of feasibility: the set of all points that satisfies ALL constraints
4. Plot the objective function
   - Set the object function equal to some quantity. Any quantity will do, although one that is evenly divisible by both coefficients is desirable
   - Determine where the line intersects each axis
     - Assume x1 = 0, solve for x2, assume x2 =0, solve for x1
     - Plot the line and place a straight edge on the line and move it parallel to find the optimal point
5. Determine which two constraints intersect the optimal point. Solve their equations simultaneously to obtain the values of the decision variables at the optimum
6. Substitute the values obtained in the previous step into the objective function to determine the value of the objective function at the optimum

Graphical Method

1. Set up objective function and constraints in mathematical format
2. Plot the constraints

- For first constraint, replace the inequality (>= or <=) with an equal sign (=)
- Determine where the constraint intersects each axis
  - Assume x1 = 0, solve for x2
  - Assume x2 = 0, solve for x1
  - Plot the line represented by the points above
- Repeat for each constraint

Glickman Electronics: Mathematical Model

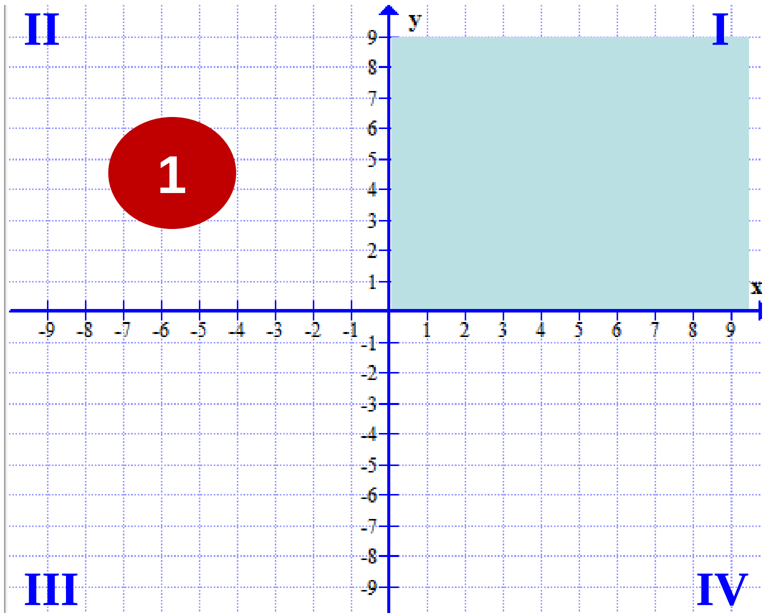
- Decision variables
  - **x1 = number of x-pods to produce**
  - **x2 = number of BlueBerrys to produce**
- Objective Function: maximize profit
  - **Max z = $7x1 + $5x2**
- Subject to (constraints):
  - **Electronic constraint: 4x1 + 3x2 ≤ 240**
  - **Assembly time constraint: 2x1 + 1x2 ≤ 100**
  - **Non-negativity constraints: x1, x2 ≥ 0**

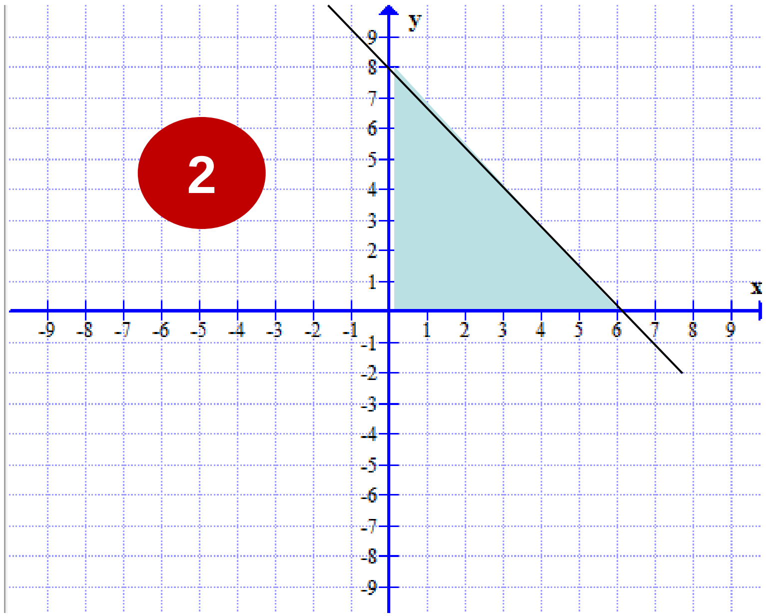
Electronic constraint:
Replace w/ equal sign: 4x1 + 3x2 = 240
x1 = 0: 4(0) + 3x2 = 240; x2 = 80
x2 = 0: 4x1 + 3(0) = 240; x1 = 60

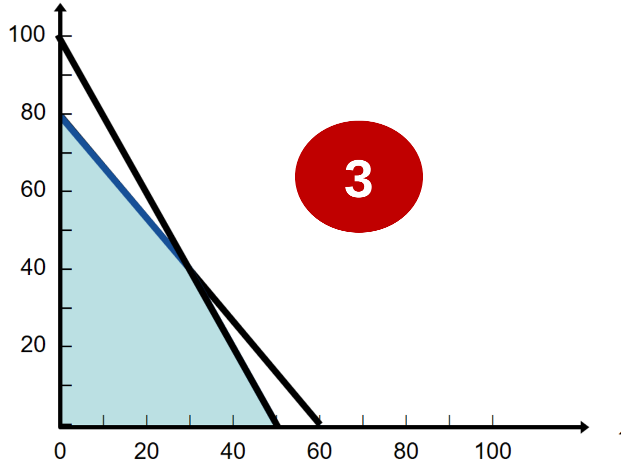
Assembly constraint:
Replace w/ equal sign: 2x1 + 1x2 = 100
x1 = 0: 2(0) + 1x2 = 100; x2 = 100
x2 = 0: 2x1 + 1(0) = 100; x1 = 50

Plot the constraints

Feasible Solution Area





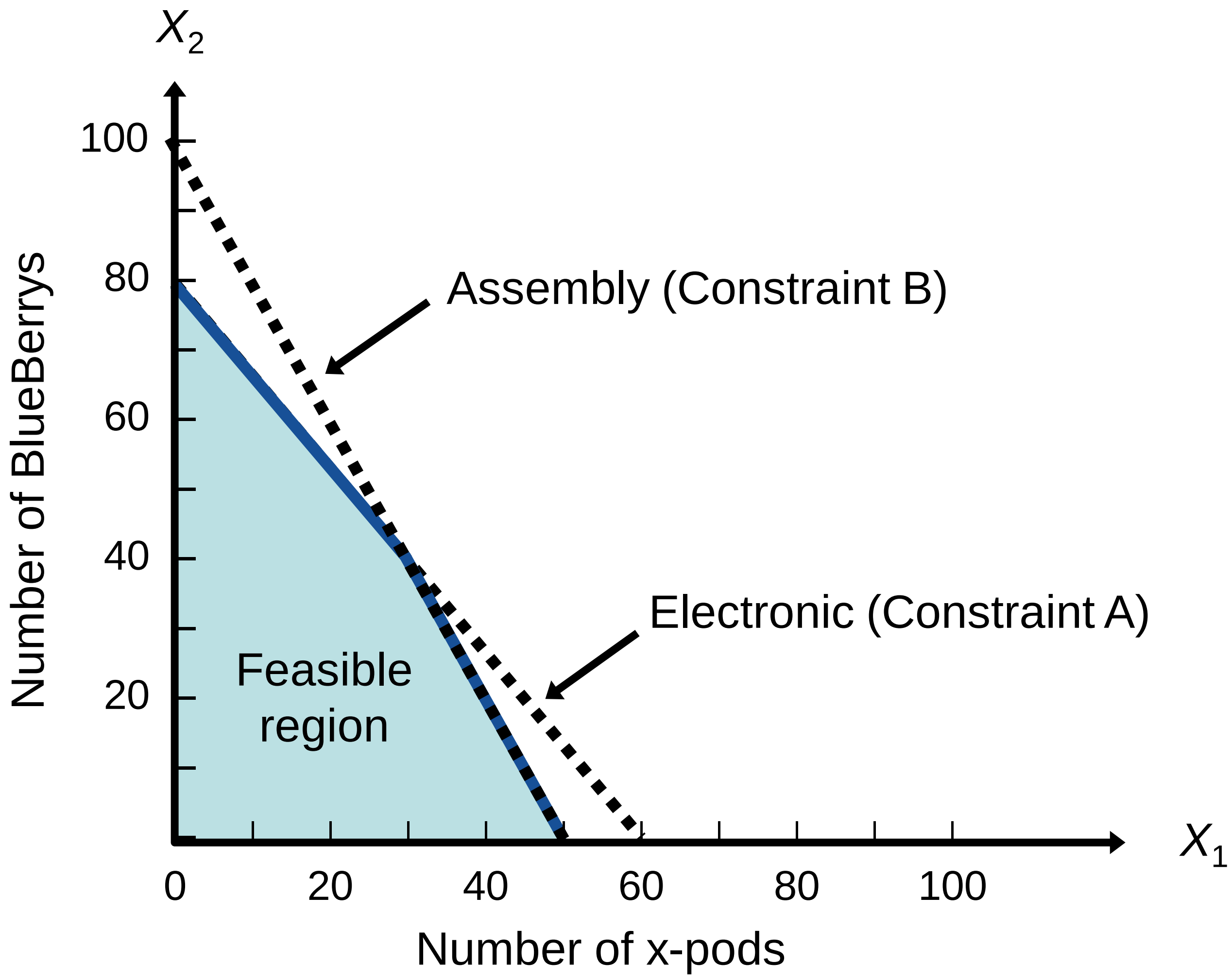


1 Non-negativity constraints

2 Plot the first constraint (electronic)

3 Plot the second constraint (assembly)

Graphical Solution



Graphical Solution

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4. Plot the objective function

- Set the object function equal to some quantity. Any quantity will do, although one that is evenly divisible by both coefficients is desirable
- Determine where the line intersects each axis
  - Assume x1 = 0, solve for x2, assume x2 =0, solve for x1
  - Plot the line and place a straight edge on the line and move it parallel to find the optimal point

Max z = 7x1 + 5x2

Guess z = 210: 210 = 7x1 + 5x2

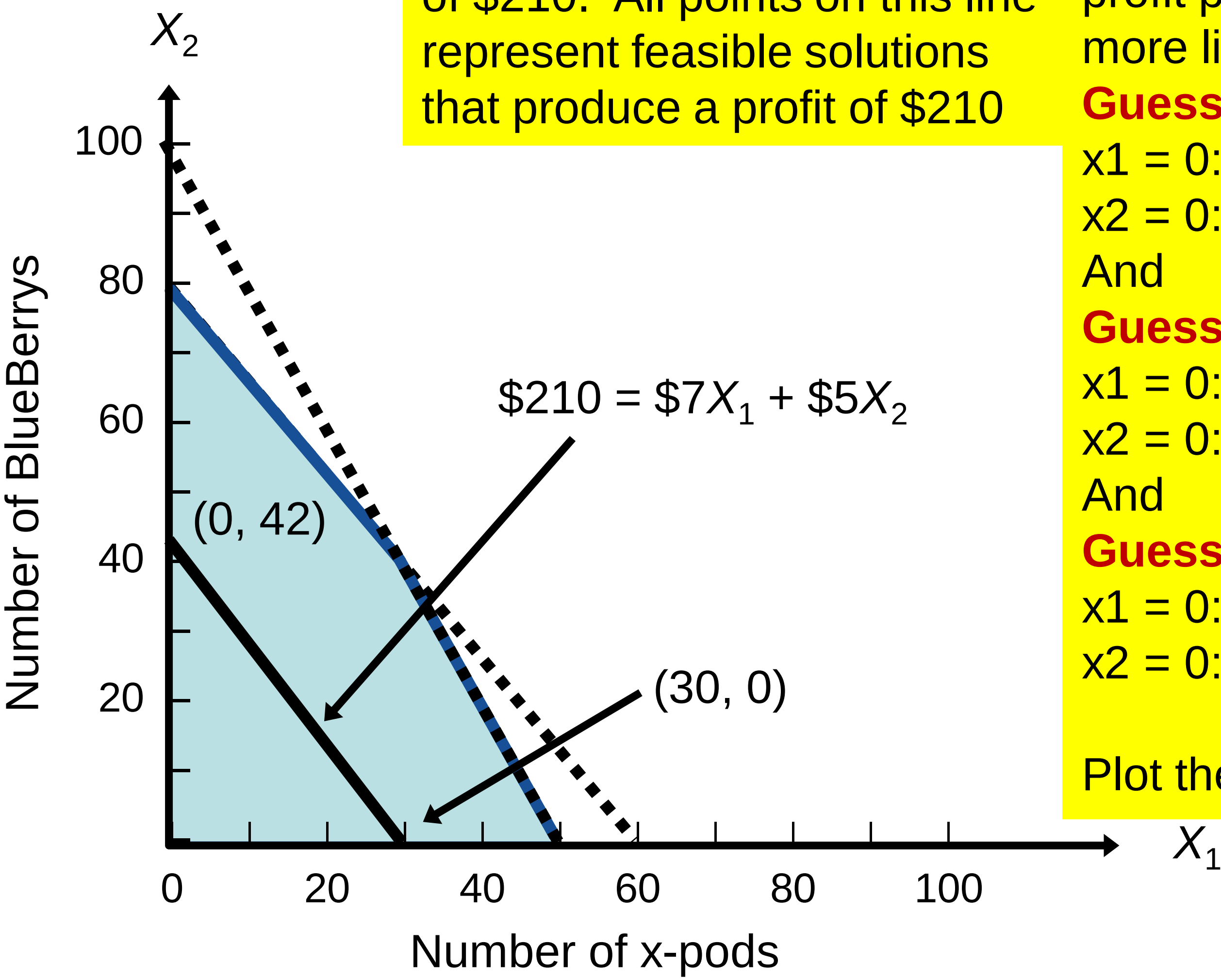
x1 = 0: 210 = 7(0) + 5x2; **x2 = 42**

x2 = 0: 210 = 7x1 + 5(0); **x1 = 30**

Plot this line

Graphical Solution

We call this line an "***iso-profit line***" that will yield a total profit of $210. All points on this line represent feasible solutions that produce a profit of $210

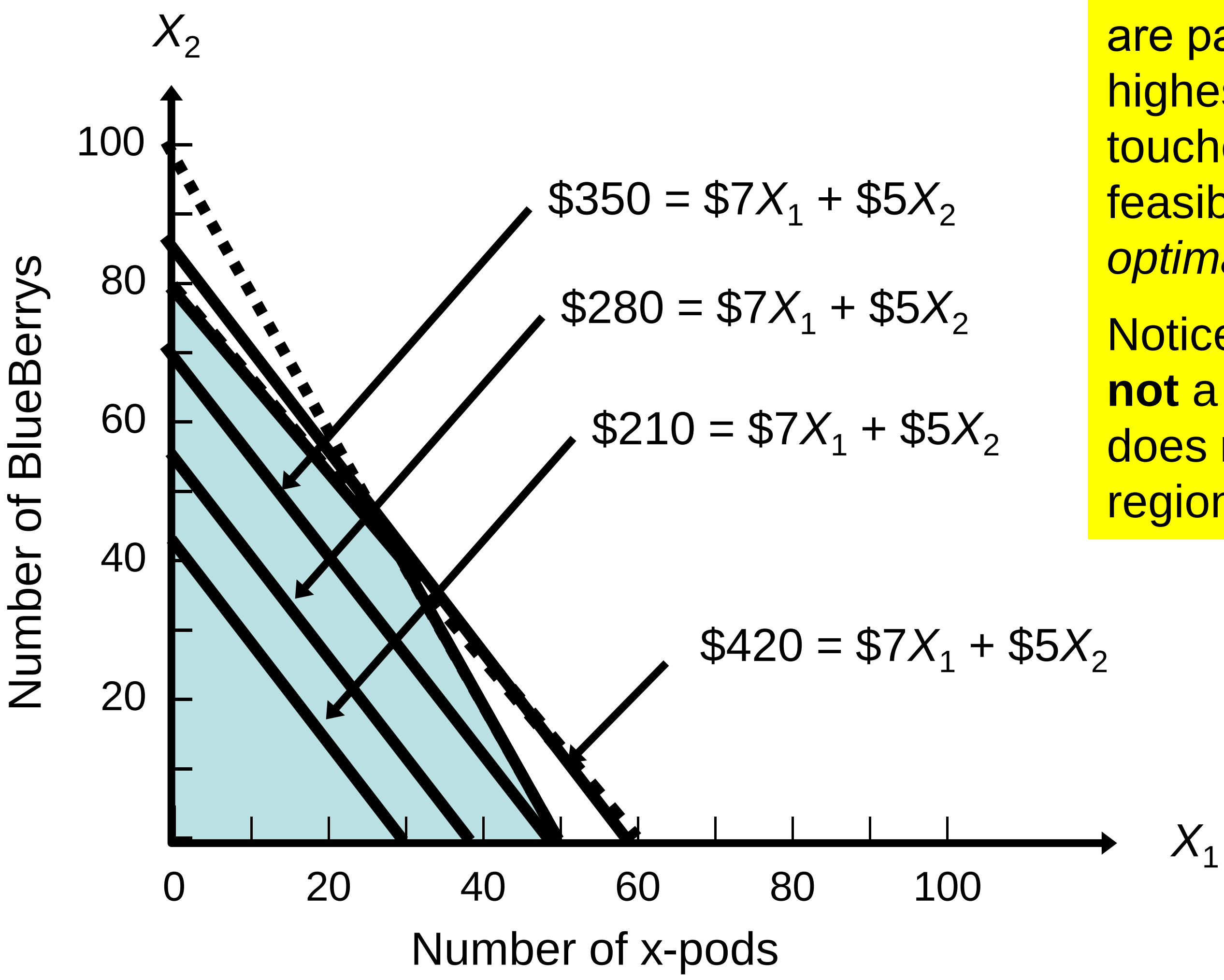


However, the iso-profit line of $210 does not yield the highest profit possible. So we will try 3 more lines:

Guess z = 280: 280 = 7x1 + 5x2
x1 = 0: 280 = 7(0) + 5x2; x2 = 56
x2 = 0: 280 = 7x1 + 5(0); x1 = 40
And
Guess z = 350: 350 = 7x1 + 5x2
x1 = 0: 350 = 7(0) + 5x2; x2 = 70
x2 = 0: 350 = 7x1 + 5(0); x1 = 50
And
Guess z = 420: 420 = 7x1 + 5x2
x1 = 0: 420 = 7(0) + 5x2; x2 = 84
x2 = 0: 420 = 7x1 + 5(0); x1 = 60

Plot these lines

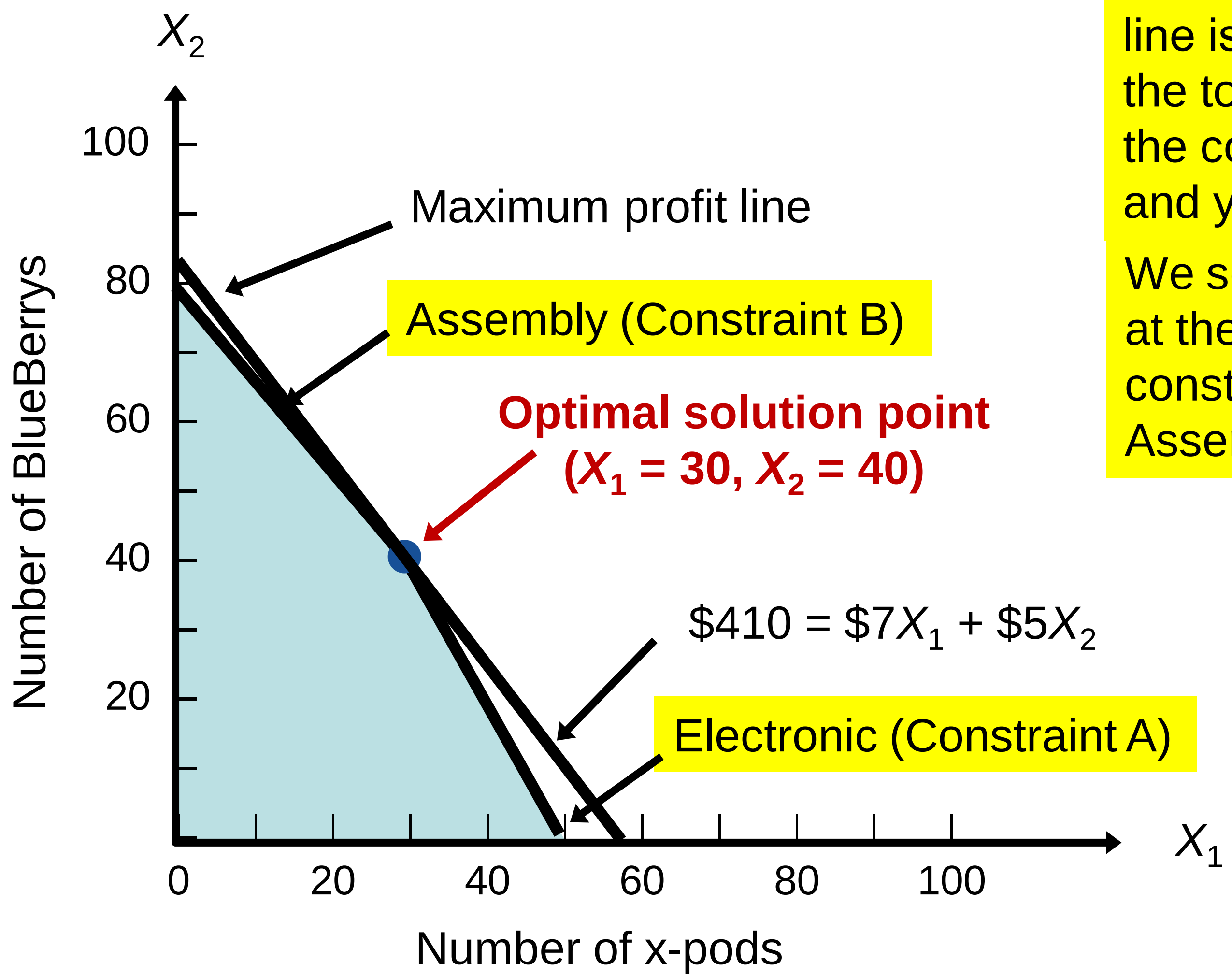
Graphical Solution



Notice that the iso-profit lines are parallel to each other….the highest iso-profit line that still touches *some point* of the feasible region will pinpoint the *optimal solution*.

Notice that the 4th line ($420) is **not** a viable solution because it does not touch the feasible region

Graphical Solution



The highest possible iso-profit line is shown here…it touches the top of the feasible region at the corner point (x1=30, x2=40) and yields a profit of $410

We see that this corner point is at the intersection of the two constraints (Electronics and Assembly)!

Graphical Solution

4. Plot the objective function
   - Set the object function equal to some quantity. Any quantity will do, although one that is evenly divisible by both coefficients is desirable
   - Determine where the line intersects each axis
     - Assume x1 = 0, solve for x2, assume x2 =0, solve for x1
     - Plot the line and place a straight edge on the line and move it parallel to find the optimal point
5. Determine which two constraints intersect the optimal point. Solve their equations simultaneously to obtain the values of the decision variables at the optimum
6. Substitute the values obtained in the previous step into the objective function to determine the value of the objective function at the optimum

Electronic constraint: 4x1 + 3x2 = 240
Assembly constraint: 2x1 + 1x2 = 100

Simultaneous Equations
Equation 1: 4x1 + 3x2 = 240
Equation 2: 2x1 + 1x2 = 100
Multiply Equation 2 by (-2) and add to Equation 1:

[4x1 + 3x2 = 240]
+ [-4x1 - 2x2 = -200]
1x2 = 40 → **x2 = 40**

Substitute x2 = 40 into either equation to solve for x1 (I've chosen Eq. 1):
4x1 + 3(x2=40) = 240
4x1 +120 = 240
4x1 = 240 – 120
4x1 = 120 → **x1 = 30**

Substitute x1 and x2 into objective function to determine the optimum value: 7(30) + 5(40) = $410

Algebraic Method

- Use simultaneous equations to determine the optimal values of the decision variables and the objective function
  - ONLY use for *two* decision variables AND *two* constraints
  - **(if more than two decision variables and constraints, use EXCEL SOLVER to solve problem!)**

- Steps:
  1. Change constraint inequalities to equalities
  2. Eliminate one unknown variable, solve for second unknown
  3. Plug solution into either equation to solve for first unknown
  4. Plug solutions into objective function and solve objective function

Algebraic Method Example

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- Use simultaneous equations to determine the optimal values of the decision variables and the objective function:
  - Maximize z = 4x1 + 3x2
  - Subject to:
    - Material: 6x1 + 4x2 <= 48 lb
    - Labor: 4x1 + 8x2 <= 80 hours

Change from inequalities: Material: 6x1 + 4x2 = 48, Labor 4x1 + 8x2 = 80

Eliminate one variable and solve for the other:
Multiply Eqtn 1 (Material) by (-2) and add to Eqtn 2 (Labor):

[-12x1 - 8x2 = -96]
+ [4x1 + 8x2 = 80]
-8x1 = -16 → **x1 = 2**

Plug x1=2 into either equation and solve for x2 (I chose Eqtn 1):

6(x1=2) + 4x2 = 48
12 + 4x2 = 48
4x2 = 36 → **x2 = 9**

Plug x1=2 and x2=9 into Objective Function: 4(x1=2) + 3(x2=9) = 8 + 27 = **35**

Slack and Surplus

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- Binding constraint: a constraint that forms the optimal corner point of the feasible solution space
- Surplus: when the optimal values of decision variables are substituted into a ≥ (greater than or equal to) constraint and the resulting value **exceeds** the right side value
  - Excess above resource requirements
- Slack: when the optimal values of decision variables are substituted into a ≤ (less than or equal to) constraint and the resulting value is **less than** the right side value
  - Unused resources

Slack and Surplus Examples

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A. One constraint: $3x1 + 2x2 \le 100$, optimal values of decision variables: x1=10, x2=20

Substitute decision variables into constraint eqtn: 3(x1=10) + 2(x2=20)

Do the math: 3(10) + 2(20) = 70…… since constraint is "<= ", 30 units "SLACK"

B. One constraint: $3x1 + 2x2 \le 100$, optimal values of decision variables : x1=20, x2=20

3(20) + 2(20) = 100…… since difference is 0, it's a "Binding Constraint"

C. One constraint: $4x1 + 1x2 \ge 50$, optimal values of decision variables : x1=10, x2=15

4(10) + 1(15) = 55…… since constraint is ">= ", 5 units "SURPLUS"

D. One constraint: $3x1 + 2x2 \ge 50$, optimal values of decision variables : x1=10, x2=10

3(10) + 2(10) = 50…… since difference is 0, it's a "Binding Constraint"

Excel Solver: Answer Report

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Microsoft Excel 14.0 Answer Report

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$D$4	Objective Function Left Hand Side	0	410

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$8	Solution Values x1 = x-pods	0	30	Contin
$C$8	Solution Values x2 = Blueberrys	0	40	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$5	Electronics Left Hand Side	240	$D$5<=$F$5	Binding	0
$D$6	Assembly Left Hand Side	100	$D$6<=$F$6	Binding	0

Note: Excel table labels "slack" (regardless if it is "slack" or "surplus")

Sensitivity Analysis

- Since business environment is dynamic, companies need to develop contingency plans for the optimal solutions

- <u>Sensitivity analysis:</u> used to assess the impact of potential changes to parameters of an LP model

- There are three types of potential changes:
  - Objective function changes
  - Right-hand-side (RHS) values of constraints
  - Constraint coefficients

Sensitivity Analysis

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- Range of optimality: the range of values for the coefficients in the objective function over which the solution of the decision variables remain the same (LHS)

- Range of feasibility: the range of values for the right-hand side (RHS) of a constraint over which the shadow price remains the same

- Shadow price: the amount by which the value of the objective function would change with a one-unit change in the RHS value of the constraint
  - Positive (negative) values indicating how much a one-unit increase (decrease) in the original amount of a constraint would improve (worsen) the optimum value Z of the objective function
  - Alternatively, if a premium price must be paid for the increase of RHS, then the shadow price represents the maximum premium (excess over the regular price) that would be worth paying
  - Shadow price is only valid when the RHS changes are within the Range of Feasibility

Excel Solver: Sensitivity Report

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Objective function coefficients:
Orig = 7, Range of Opt: 6.67 – 10
Orig = 5, Range of Opt: 3.5 – 5.25

Microsoft Excel 14.0 Sensitivity Report

Range of Optimality

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$8	Solution Values x1 = x-pods	30	0	7	3	0.333333333
$C$8	Solution Values x2 = Blueberrys	40	0	5	0.25	1.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$5	Electronics Left Hand Side	240	1.5	240	60	40
$D$6	Assembly Left Hand Side	100	0.5	100	20	20

Glickman Electronics: Mathematical Model

- Decision variables
  - x1 = number of x-pods to produce
  - x2 = number of BlueBerrys to produce
- Objective Function: maximize profit
  - Max z = $7x1 + $5x2
- Subject to (constraints):
  - Electronic constraint: 4x1 + 3x2 ≤ 240
  - Assembly time constraint: 2x1 + 1x2 ≤ 100
  - Non-negativity constraints: x1, x2 ≥ 0

Shadow Price

Range of Feasibility

Constraint RHS:
Orig = 240, Range of Feas: 200 – 300
Orig = 100, Range of Feas: 80 - 120

Computer Profit Example

A firm that assembles computers and computer equipment is about to start production of two new types of microcomputers. Each type will require assembly time, inspection time and storage space. The amounts of each of these resources that can be dedicated to the production of the microcomputers is limited. The manager of the firm would like to determine the quantity of each microcomputer to produce in order to maximize the profit generated by sales of these microcomputers.

	Type 1	Type 2
Profit per unit	$60	$50
Assembly time per unit	4 hours	10 hours
Inspection time per unit	2 hours	1 hour
Storage space per unit	3 cubic feet	3 cubic feet

Resource	Amount Available
Assembly time	100 hours
Inspection time	22 hours
Storage space	39 cubic feet

Notice units:
Constraints can be mixed units but Objective Function Coefficients MUST be same units!!!

Computer Profit Example

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- Decision Variables:

  x1 = number of Type 1 computers to manufacture
  x2 = number of Type 2 computers to manufacture

- Objective Function: maximize profit

  Maximize $Z = 60x1 + 50x2$

- Subject to (constraints):

Assembly	$4x1 + 10x2 \leq 100$ hours
Inspection	$2x1 + 1x2 \leq 22$ hours
Storage	$3x1 + 3x2 \leq 39$ cubic feet
Non-negativity	$x1, x2 \geq 0$

- Now, input data into spreadsheet and use Excel solver to find the solution, then select Answer Report and Sensitivity Report →

Computer Profit Example: Answer Report

Microsoft Excel 11.0 Answer Report
Worksheet: Computer Profit Input
Report Created:

Target Cell (Max)

Cell	Name	Original Value	Final Value
$B$19	Profit x1 = # type 1	$0.00	$740.00

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$12	Amount produced x1 = # type 1	0	9
$C$12	Amount produced x2 = # type 2	0	4

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$15	Assembly x1 = # type 1	76	$B$15<=$E$6	Not Binding	24
$B$16	Inspection x1 = # type 1	22	$B$16<=$E$7	Binding	0
$B$17	Storage x1 = # type 1	39	$B$17<=$E$8	Binding	0

Computer Profit Example: Sensitivity Report

Operations & Supply Chain University

Microsoft Excel 11.0 Sensitivity Report
Worksheet: Computer Profit Input
Report Created:

Range of Optimality

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$12	Amount produced x1 = # type 1	9	0	60	40	10
$C$12	Amount produced x2 = # type 2	4	0	50	10	20

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$15	Assembly x1 = # type 1	76	0	100	1E+30	24
$B$16	Inspection x1 = # type 1	22	10	22	4	4
$B$17	Storage x1 = # type 1	39	13.33333333	39	4.5	6

Shadow Price

Range of Feasibility

1E+30 = infinity → meaning, even an infinite increase could not improve the solution!

Computer Profit Example: Sensitivity Analysis - Solution

Operations & Supply Chain University

- Conduct a “what-if” (sensitivity) analysis on the computer example:
  1. What is the optimal profit and optimal product mix?

  2. Suppose 1 additional unit of capacity becomes available. How will this affect the optimal profit
     - An extra hour of inspection time?

     - An extra hour of assembly time?

     - An extra cubic foot of storage space?

Computer Profit Example: Answer Report

Operations & Supply Chain University

Microsoft Excel 11.0 Answer Report
Worksheet: Computer Profit Input
Report Created:

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     Product mix: 9 units of Type 1 computer should be produced
     4 units of Type 2 computer should be produced
     Optimal profit is $740

  2. Suppose 1 additional unit of capacity becomes available. How will this affect the optimal profit
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Computer Profit Example: Sensitivity Report

Operations & Supply Chain University

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Worksheet: Computer Profit Input
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Range of Optimality

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Range of Feasibility

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Operations & Supply Chain University

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     Optimum profit is $740

  2. Suppose 1 additional unit of capacity becomes available. How will this affect the optimal profit
     - An extra hour of inspection time?
       - Profit would increase by $10 → 740 + 10 = $750
     - An extra hour of assembly time?
     - An extra cubic foot of storage space?

Computer Profit Example: Sensitivity Report

Microsoft Excel 11.0 Sensitivity Report
Worksheet: Computer Profit Input
Report Created:

Range of Optimality

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
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     - An extra cubic foot of storage space?

Computer Profit Example: Sensitivity Report

Operations & Supply Chain University

Microsoft Excel 11.0 Sensitivity Report
Worksheet: Computer Profit Input
Report Created:

Range of Optimality

Adjustable Cells

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Range of Feasibility

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Operations & Supply Chain University

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  2. Suppose 1 additional unit of capacity becomes available. How will this affect the optimal profit
     - An extra hour of inspection time?
       - Profit would increase by $10 → 740 + 10 = $750
     - An extra hour of assembly time?
       - No impact to profit
     - An extra cubic foot of storage space?
       - Profit would increase by $13.33 → 740 + 13.33 = $753.33

Computer Profit Example: Sensitivity Analysis - Solution

Operations & Supply Chain University

- Conduct a “what-if” (sensitivity) analysis on the computer example:
  3. Suppose the profitability of the Type 1 Computer increases by $10?

  4. Suppose the profitability of the Type 2 Computer decreases by $15?

  5. If an additional 3 inspection hours can be obtained by paying a $2/hour premium, what will happen to company profit?

Computer Profit Example: Sensitivity Report

Operations & Supply Chain University

Microsoft Excel 11.0 Sensitivity Report
Worksheet: Computer Profit Input
Report Created:

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
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Range of Optimality

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Shadow Price

Range of Feasibility

Computer Profit Example: Sensitivity Analysis - Solution

Operations & Supply Chain University

- Conduct a "what-if" (sensitivity) analysis on the computer example:

3. Suppose the profitability of the Type 1 Computer increases by $10?

   9x10 = 90. Profit would increase by $90 → 740 + 90 = $830

4. Suppose the profitability of the Type 2 Computer decreases by $15?

5. If an additional 3 inspection hours can be obtained by paying a $2/hour premium, what will happen to company profit?

Computer Profit Example: Sensitivity Report

Operations & Supply Chain University

Microsoft Excel 11.0 Sensitivity Report
Worksheet: Computer Profit Input
Report Created:

Range of Optimality

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
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Range of Feasibility

Computer Profit Example: Sensitivity Analysis - Solution

- Conduct a "what-if" (sensitivity) analysis on the computer example:
  3. Suppose the profitability of the Type 1 Computer increases by $10?

     9x10 = 90. Profit would increase by $90 → 740 + 90 = $830

  4. Suppose the profitability of the Type 2 Computer decreases by $15?

     4 x15 = 60. Profit will decrease by $60 → 740 – 60 = $680

  5. If an additional 3 inspection hours can be obtained by paying a $2/hour premium, what will happen to company profit?

Computer Profit Example: Sensitivity Report

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Report Created:

Range of Optimality

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     4 x15 = 60. Profit will decrease by \$60 → 740 – 60 = \$680

  5. If an additional 3 inspection hours can be obtained by paying a \$2/hour premium, what will happen to company profit?

     Inspection shadow price is \$10/hour. But must pay premium (3x\$10 Shadow Price) – (3x\$2 Premium) → 30 – 6 = 24.
     Profit would increase by → 740 + 24 = \$764